A TWO-ZONE MODEL FOR LONGITUDINAL DISPERSION IN CHANNELS WITH IDEALIZED POOLS AND RIFFLES

ERIC M. VALENTINE, ZULFIQAR ALI AND DAVID C. SWAILES

A one-dimensional two-zone mathematical model, comprising a pair of advection-dispersion equations coupled by a mass exchange term, is proposed to study longitudinal dispersion in channels with sequences of pools and riffles. An implicit finite-difference numerical scheme is employed, and its effectiveness is assessed with reference to known analytical solutions. Moreover, sets of longitudinal dispersion experiments were performed on various simple geometries of sequences of pools and riffles developed in a laboratory flume. The results were compared with corresponding numerical solutions to calibrate the two-zone model.

KEY WORDS: Dispersion, Open-Channel Flow, Dead Zones, Pools and Riffles, Numerical Model.


Pro studium podélné disperze v korytech s opakující se soustavou tuní a prahu byl navržen jednorozmerný dvouzónový matematický model. Model zahrnuje dvojici rovnic pro advektivní disperzi doplnených výrazem pro prenos hmoty. Byl použit implicitní model konečných diferenčí a jeho vhodnost overena porovnáním se známým analytickým rešením. Navíc, v laboratorním žlabu byla provedena série merení podélné disperze pro ružně jednoduché geometrie koryta se strídajícími se tunemi a prahy. Pro kalibraci dvouzónového modelu byly výsledky merení porovnány s odpovídajícími matematickými rešeními.

KLÍCOVÁ SLOVA: disperze, proudení v otevřených korytech, mrtvé zóny, tune a prahy, numerický model.

Eric M. Valentine, Senior Lecturer, Dept. of Civil Engineering, Univ. of Newcastle upon Tyne, NE1 7RU, UK.
Zulfiqar Ali, Lecturer, Technical University of Lahore, Pakistan.
David C. Swailes, Lecturer, Dept. of Engineering Mathematics, Univ. of Newcastle upon Tyne, NE1 7RU, UK.
1. Introduction

It is widely acknowledged that simple Fickian models do not provide accurate descriptions of the longitudinal transport of contaminants in natural channels, especially during early stages of the mixing process (Young and Wallis, 1993). Even after the initial period defined by Taylor (1954) skewed concentration-time profiles and nonlinear growth of variance of the contaminant clouds have been observed (Day, 1975). Hays (1966) developed a dead zone model to explain the long tails observed in concentration-time profiles. Subsequently, Thackston and Schnelle (1970), Valentine and Wood (1977, 1979a, 1979b), Benca and Walters (1983), Chikwendu and Ojiakor (1985) and others developed models to explain mixing processes in natural channels.

In line with earlier works, [see for example Smith (1976, 1979), Benca and Walters (1983), Chikwendu and Ojiakor (1985), Seo (1990a, 1990b), Seo and Maxwell (1992)] and considering flow characteristics of pools and riffles (Miller and Wenzel, 1984, 1985), a one-dimensional two-zone model consisting of flow and slow zones is proposed. The two-zone model is applied to study longitudinal dispersion in channels with sequences of pools and riffles. This model is a more general form of the dead zone mechanism having advection and dispersion terms in both zones along with the dead zone parameters. The advection and dispersion terms of the slow zone simplify the calculation of dead zone parameters with varying discharges. The proposed model could be of value for channels with pools and riffles. Such channel flows are highly turbulent, with large variations in flow velocities and cross-sectional areas (Benca and Walters, 1983; Bhowmik and Demissie, 1982; Miller and Wenzel, 1985). The near-bed velocity of pools also increases progressively with the increase in flow rate (Sear, 1996).

As in Fig. 1, the flow is divided into two zones: a flow zone (zone 1) and a boundary slow zone (zone 2). The flow zone is an upper region of water having larger flow velocities compared to the slow zone, which is mainly controlled by surface roughness and pools. This two-zone model was studied numerically using an implicit-finite difference method. The numerical results were assessed with reference to analytical solutions for uniform flows (Chikwendu and Ojiakor, 1985) with a constant dispersion coefficient. Computer programs were developed for the numerical and the analytical solutions of the model. The programs computed concentration variations, displacement, velocity, spatial variance and total mass in the flow and the slow zones.
To create a reliable experimental database and to validate and calibrate the proposed two-zone dispersion model, hydraulic and longitudinal dispersion experiments for a variety of flow rates were performed on four idealized geometries of six pool-riffle sequences constructed in a laboratory flume. Laboratory experiments were also performed to measure mass exchange coefficients between the flow and slow zones.

The aims of this research were:

a) to study longitudinal dispersion and the hydraulics of channels with sequences of pools and riffles.
b) to test and validate the numerical scheme by reference to known analytical solutions obtained for special cases of the general model, and
c) to calibrate the proposed model parameters with the experimental data.

2. The transport model and numerical method

In this paper, the transport of neutrally buoyant solute material introduced into a fully developed turbulent flow in an open-channel is considered. In particular, attention is given to longitudinal transport in channels that possess bed storage zones (pools) along their length (Fig. 1). Stretches between pools (the riffles) exhibit significant surface roughness and typically have a relatively steep gradient compared to the pools.

Fig. 1. Sketch of pools and riffles in a natural channel, showing average velocity and concentration in flow and slow zones.


In this paper, the study is confined to slow zones adjoining the channel bed, but trapping may occur due to slow zones associated with channel sides. In
principle, however, the model, with suitable reinterpretation, could be applied to flows with side slow zones.

Denoting the cross-sectional average solute concentration in zone \( n \) at the longitudinal spatial co-ordinate \( x \) at time \( t \) by \( C_n(x,t) \), the evolution and interaction of these concentrations is modelled via two advection-dispersion equations, coupled by a mass exchange term:

\[
A_n \frac{\partial}{\partial x} C_n - \frac{\partial}{\partial x} U_n A_n C_n - \frac{\partial}{\partial x} D_n A_n \frac{\partial}{\partial x} C_n = m \frac{\partial}{\partial x} C_n \quad n = 1, 2. \tag{1}
\]

In this equation, \( A_n(x) \) denotes the cross-sectional area \([\text{m}^2]\) of the flow in zone \( n \) at the point \( x \). Similarly \( U_n(x) \) and \( D_n(x) \) denote, respectively, the mean fluid velocity \([\text{m sec}^{-1}]\) and longitudinal dispersion coefficient \([\text{m}^2 \text{sec}^{-1}]\) in zone \( n \). In the final term on the right-hand side of Equat. (1) \( C_2 - C_1 \), and \( m \) denotes the mass exchange coefficient per unit length \([\text{m}^2 \text{sec}^{-1}]\) at the interface of two zones. Hence, the sign preceding this last term is positive for \( n = 1 \) and negative for \( n = 2 \).

It may be noted that the model expressed by Equat. (1) constitutes a generalization of the two zone mechanism and allows for the variations in hydraulic parameters in both zones, and also for advective and dispersive transport in the slow zone. If we assume average slow zone velocity to be zero and neglect the dispersion term then this two-zone model reduces to a simple one-dimensional advection-dispersion model with dead zone. This two zone mechanism also converges to a classical Fickian type advection-dispersion equation when the slow zone area diminishes. The experimental results presented below indicate that longitudinal advection could be a significant transport mechanism in the slow zones, and could be accounted for in any dispersion model.

Assuming, at \( t = 0 \), a mass \( M \) of solute is introduced uniformly across the flow area at \( x = 0 \), the corresponding initial conditions for Equat. (1) are

\[
C_n(x,0) = c_0(x) \quad n = 1, 2, \tag{2}
\]

where \( c_0 = M/(A_1(0) + A_2(0)) \) at the \( x = 0 \), and \( ?(x) \) is the Dirac delta function.

This initial condition represents an idealization of a localised, nearly instantaneous injection of solution (at \( x = 0, t = 0 \)) which is uniformly distributed over both zones. This was the situation that was aimed at in the laboratory experiments.

Appropriate far field conditions for Equat. (1) are then

\[
C_n(x,t) = 0 \quad \text{as } |x| \to \infty, \quad t > 0 \quad n = 1, 2. \tag{3}
\]
Equat. (1), together with (2) and (3), represents a model for the dispersion of solution clouds as quantified by the cross-sectionally averaged concentrations $C_1$ and $C_2$. Since analytic solutions to this general model are not known the model has been studied numerically as described below. In the following section the performance of the numerical approach is assessed by reference to, albeit unrealistic, known solutions of a special case of the general model. Agreement of the numerical results with these analytic solutions supports the suitability of the numerical method which is then applied to more general (and physically relevant) cases.

The following implicit finite-difference numerical approach is employed to study the model given by Equat. (1), (2), and (3). Uniformly spaced grid points $(x_i, t_k)$, $i = 0, 1, ..., N$, $k = 0, 1, ..., K$, are introduced, where $\Delta x$ and $\Delta t$ define distance and time steps respectively, and the integers $K$, $M$ and $N$ define the domain over which the computation is to be performed.

Following Stone and Brian (1963) the grid point values of the concentration time derivatives are approximated using

$$\frac{?}{?t} C_i^k \approx \frac{1}{6} C_i^{k+1} + \frac{2}{3} C_i^k - \frac{1}{6} C_i^{k-1},$$

where $C(x_i, t_k)$, $C_i^k$ ($C$ is either $C_1$ or $C_2$), and where $?^t$ is the forward difference operator, $?^t C_i^k = (C_i^{k+1} - C_i^k) / \Delta t$.

To accommodate gradients generated by variations of cross-sectional area of flow and slow zones in channels with pools and riffles, it is appropriate to split the spatial derivatives of $U A_n$ and $D A_n$ from that of $C_n$ on the right-hand side of Equat. (1): So for example, writing $Q = U_n A_n$, $\frac{?}{?x} (Q C) \approx \frac{Q_i}{\Delta x} C_i^k - \frac{Q}{\Delta x} C_i^k$ and the gradients are then approximated by

$$\frac{?}{?x} C_i^k \approx \frac{1}{2} \delta_x C_i^{k+1} - \delta_x C_i^k \quad \text{and} \quad \frac{?}{?x} Q_i = \delta_x Q_i,$$

where $\delta_x$ and $\delta_x^t$ are the central and forward difference operators respectively; $\delta_x^t C_i^k = (C_i^{k+1} - C_i^{k-1}) / 2 \Delta x$ and $\delta_x Q_i = (Q_i^{k+1} - Q_i^{k-1}) / 2 \Delta x$. A similar procedure is adopted for the dispersive terms, and the standard Crank-Nicholson approximation is used for $?^2 C / ?x^2$. Following Seo (1990a) the mass exchange term of Equat. (1) is discretized as

$$\delta_{x,t} C_i^k \approx C_{i+1}^k - C_{i-1}^k.$$
Corresponding to Equat. (2) the initial condition for the numerical scheme is $C_{n,0}^0 = c_n^0$, and corresponding to Equat. (3) the boundary conditions are $C_{n,M}^k = C_{n,N}^k = 0$, $k \neq 0$.

The resulting system of linear equations for $C_n^k(x, t_k)$ is written in the following block matrix form

$$
\begin{pmatrix}
?A^{(1)} & ?B^{(1)} & ?P^{(1)} & 0 \\
?A^{(2)} & ?B^{(2)} & ?P^{(2)} & 0
\end{pmatrix}
\begin{pmatrix}
?\xi^{(1)}_1 \\
?\xi^{(1)}_2 \\
?\xi^{(2)}_1 \\
?\xi^{(2)}_2
\end{pmatrix}
= \begin{pmatrix}
0 \\
0
\end{pmatrix},
$$

where $\xi^{(k)}_n = (C_{n,M}^k, \ldots, C_{n,N}^k)^T$. The blocks $A^{(i)}$, $P^{(i)}$ are tridiagonal matrices, and the $B^{(i)}$ are diagonal matrices. We omit the algebraic details of the construction of (8), which are straight-forward. The linear system given by (8) is solved using the following block Gauss-Seidel iterative scheme at each time-step:

$$
A^{(1)}\xi^{(1)}_1 = P^{(1)}\xi^{(1)}_2 + B^{(1)}\xi^{(1)}_2, \\
A^{(2)}\xi^{(2)}_2 = P^{(2)}\xi^{(2)}_1 + B^{(2)}\xi^{(2)}_1,
$$

where $\xi^{(i)}_n$ is the $i$-th iteration approximation to $\xi^{(k)}_n$ (with $\xi^{(0)}_n = \xi^{(k)}_n$). Since $A^{(1)}$ and $A^{(2)}$ are constant, tri-diagonal matrices, the solution of Equats. (9) and (10) are easily obtained, making use of upper and lower triangular decomposition of $A^{(1)}$ and $A^{(2)}$ (Press et al., 1992).

Before using this numerical approach to study the two-zone model for geometries of interest it is important to assess the effectiveness of the method itself. This is presented in the following section, making use of known analytical solutions.

3. Comparison of numerical and analytical solutions

With $D_1 \neq D_2 \neq D$, $U_n$ and $A_n$ independent of $x$, the model given by Equat. (1) simplifies to

$$\frac{?}{?t} C_n + \frac{?}{?x} U_n C_n + \frac{?^2}{?x^2} C_n + \frac{?}{?m} (A_1 - A_2) - a \text{ normalized mass exchange coefficient and} \ \frac{?}{?A_n} \frac{(A_1 - A_2)}{A_n} \text{ is the reciprocal of the normalized cross-sectional area of zone } n,$$
Equat. (11) together with the conditions given by expressions (2) and (3) have an analytical solution, (see Chikwendu and Ojiakor, 1985). The efficiency and effectiveness of the numerical approach described in the previous section has been assessed by reference to this solution. Figs. 2 and 3 show results that demonstrate the ability of the numerical method to accurately resolve both spatial and temporal variations in concentration profiles. Fig. 2 shows the time variation of the sectional average concentration \( C_0 \) at four fixed locations, while Fig. 3 shows the spatial variation of \( C_0 \) at five time points. Here \( \mu_n \) is the normalized area of zone \( n \).

The flow parameters used to obtain these results were \( U_1 = 1 \), \( U_2 = 0.1 \), \( D = 0.001 \), \( A_1 = 0.75 \), \( A_2 = 0.25 \). (These values are not typical of real flows but were used to demonstrate the capability of the numerical method to resolve the large gradients observed in these solutions). Values of the numerical parameters used to produce the results of Figs. 2 and 3 were \( \Delta x = 0.5 \) and \( \Delta t = 0.3 \).

Figs. 4 and 5 show how the first two temporal moments of the concentration \( C_0 \) vary with position: Fig. 4 depicts the mean travel time
for a range of values of the velocity ratio $U_2 / U_1$, and Fig. 5 shows corresponding results of the temporal variance.

\[ T(x) = \frac{\int_0^T \int_0^T C_0(x,t) dt}{\int_0^T C_0(x,t) dt} \quad (12) \]

Fig. 3. Analytical and numerical concentration-distance profiles.

Obr. 3. Srovnání analytického a numerického rešení závislosti prumerné koncentrace na vzdálenosti v daném case; Distance – vzdálenost.

\[ T^2(x) = \frac{\int_0^T (t - T)^2 C_0(x,t) dt}{\int_0^T C_0(x,t) dt} \quad (13) \]

The parameter values used to compute these results were $U_1 = 1$, $U_2 / U_1 = 0.25, 0.5, 0.75$, $A_1 = 0.60$, $A_2 = 0.40$, $D = 1$, $m = 0.001$. Again there is good agreement between numerical and analytical results. These results (Figs. 4 & 5) also demonstrate the significance of the velocity ratio ($U_2 / U_1$) to longitudinal dispersion. The mean travel time and temporal variance progressively increased with the decrease in velocity ratio. In the following
section the numerical results of the model are compared to the laboratory experimental data.

Fig. 4. Analytical and numerical mean travel time.
Obr. 4. Srovnání analytického a numerického rešení strední doby postupu; Mean travel time – strední doba postupu.

Fig. 5. Analytical and numerical temporal variance.
Obr. 5. Srovnání analytický a numericky urcené hodnoty casové variace pomerné rychlosti; Temporal variance – casová variace, Distance – vzdálenost.
4. Experimental studies

4.1 Laboratory apparatus

The laboratory experiments were carried out in a flume with a rectangular cross-section, 12m long, 0.31m wide, 0.45m deep with painted steel bed and glass side walls. The flume was placed on a steel skeletal frame, supported on a hinge and roller supports at the ends. A motor-operated jack system, placed beneath the roller support, enabled slope variation of the flume from $-1.67\%$ to $+1.67\%$.

The source of water to the flume was the laboratory sump via a constant head storage tank placed at an elevation of eleven meters above the laboratory floor, forming the main reservoir in the recirculation system. Two pipelines, 100mm in diameter were used to supply water at the upstream end of the flume. Discharge of water in the flume was measured using a volumetric tank at the end of the flume. Flow depth was measured with a movable standard point gauge. Vertical and transverse velocity profiles were measured using a miniature current meter with a low speed probe. An automated data acquisition and digitization system consisting of six conductivity meters, a microcomputer and an analogue to digital converter was used to record concentration-time variations. The conductivity probes were made of thin stainless steel wires, and placed at half the flow depth at chosen longitudinal locations.

In these experiments neutrally buoyant sodium chloride and methanol solution was used as a tracer. The tracer solution temperature was always brought to the flume water level by immersing the solution container in a water tank, connected to the water supply pipe.

4.2 Bed form geometries and the experimental plan

To calibrate and validate the proposed two-zone dispersion model, and to develop an improved understanding of hydraulics of channels with pools and riffles, laboratory experiments were performed on four idealized geometries of six pool-riffle sequences. Rectangular concrete blocks were used to model pools and riffles, and the transitions between the pools and the riffles were discontinuous steps. These geometries of pools and riffles were used as they do allow investigation of the relative importance of the various transport mechanism, and also provide a simple system for validating the numerical results. The dimensions of these pools and riffles were selected following a literature review of typical natural channels (Keller and Melhorn, 1978; Bhomik and Dessimie, 1982; Miller and Wenzel, 1985; Whittaker and Jaeggi, 1982) and after undertaking physical
observations of a local small stream, which has established sequences of pools and riffles.

Four sets of laboratory experiments were conducted on four geometries of pools and riffles respectively. Each set of experiments consisted of four series of hydraulic and dispersion studies at different flow rates. These experiments were performed primarily to investigate the effect of slow zones on longitudinal dispersion. A secondary aim was to study the hydraulics of channels with pools and riffles. For Set 1, a pool depth of 0.06m was modelled, while for Set 2 the depth was reduced to 0.035m keeping other dimensions constant, see Fig. 6. The surfaces of the idealized riffles for Set 1 and Set 2 experiments were smooth.

A third set of laboratory experiments was performed, in which subsidiary slow zones on the riffles were developed to simulate secondary dead zones, by fixing four 0.01m x 0.01m wooden strips at equal distances, extending across the entire width of the channel. The Set 4 experiments were performed by fixing plywood sheets equal in thickness to the roughness on the riffles, in the pools. In all the laboratory experiments the flume bed slope was the same (0.625%). All flow conditions were subcritical. Fig. 6 shows the geometries used to perform the laboratory experiments.

![Fig. 6. Geometries of idealized pools and riffles modelled in the laboratory flume.](image)
4.3 Hydraulic studies

To study the hydraulics of channels with pools and riffles and to provide parameter values for the two-zone transport model, depths of flow and velocity profiles were measured. Water surface and bed profiles were measured along the centreline of the flume. Vertical velocity profiles were measured at stations S1 to S5, as shown in Fig. 7, while transverse velocity profiles at half the flow depths were measured at S3, S4 and S5, in a typical pool-riffle sequence.

![Fig. 7. Velocity measurement stations in a typical pool-riffle sequence.](image)

Vertical velocity profiles were almost uniform over smooth riffles, while gradients in the velocity profiles were observed over the riffles with additional roughness. Steep gradients in the vertical velocity profiles were found at S2, which was 0.125m from the upstream end of the pool. Recirculation zones were observed at the upstream end of these idealized pools of Sets 1, 3 and 4 for all flow rates. No recirculation of water was noted in Set 2 experiments. The magnitude of velocities decreased downstream of the pools with an increase in flow depths. Average slow zone velocities in the pools also increased progressively with the increase in discharge for all data sets. The vertical velocity profiles for Set 2 experiments are shown in Fig. 8.

In the measurement of the velocity fluctuations a considerable amount of turbulence was observed. At the start of the pools the profiles did not follow the conventional logarithmic shape. At S4 and S5 the velocity profiles were fairly uniform with less turbulence. The flow in the laboratory flume was turbulent (Reynolds number varied from 4200–15000 for the flow rates studied) and non-uniform with large fluctuations in the velocity values.

Channel bed and water surface profiles were measured along the centreline of the flume. Water surface profiles followed similar patterns over the pools and the riffles for Set 1 and Set 2 experiments. The water level increased downstream in the pools for all flow rates. The effect of additional roughness on the riffles was significant and after each roughness element, a drop in water level with an increase in the velocity was found. The average flow depth over the riffles increased considerably due to the additional roughness elements.
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5. Comparison of experimental and numerical solutions

Four sets of longitudinal dispersion experiments, each comprising a series of systematic laboratory studies were performed to validate the parameters of the two-zone model. In this paper only the results of two series of Sets 1 and 2 are presented.

To assess the individual effects of pools and riffles, concentration-time profiles were recorded at 1.4m and 0.6m intervals, corresponding to the upstream and downstream end of the riffles. The data were recorded at eleven stations, injecting a known quantity of the tracer solution at the upstream of the first riffle as a line source across the flume. Several dispersion runs were repeated using the same quantity of tracer solution with the same hydraulic
conditions. Average concentration-time profiles were used to compute the statistical parameters, mean travel time, temporal variance, skew, mean velocity and longitudinal dispersion coefficients, which were compared with the simulated values.

Tab. 1 presents the input values of the hydraulic parameters, vertical velocities, flow depths and the discharge, used in the two-zone model. Flow depths were measured at a number of points along the centerline of the flume. Slow zone depths were measured from the bottom of the flume to the top of the roughness on the riffles. Velocities in flow and slow zones in the pools were computed by averaging the vertical velocity profiles measured at various stations. Flow and slow zone velocities on the riffles were computed in a similar fashion. There was a considerable increase in average slow zone velocities with the increase in flow rates for all data sets.

The values of the mass exchange coefficients used in numerical simulations are presented in Tab. 2, which were approximately the same as those measured experimentally for Geometry 1-5 (Appendix Fig. A1). The non-dimensional simulated exchange coefficient $k^*$ varied from 0.027 to 0.0315. These values were higher than the non-dimensional value 0.02 proposed by Valentine and Wood (1977) and lower than the values reported by Seo (1990a) and the values computed in this study for higher pool lengths (Appendix). This variation emerged due to advection and dispersion terms employed in the slow zone of the two-zone model.

Various empirical relations for longitudinal dispersion coefficients for open channel flow have been developed considering vertical and transverse velocity variations and turbulence (Elder, 1959; Fischer, 1967; Chikwendu and Ojiakor, 1985). However, the dispersion coefficients employed in this two-zone model were for uniform open channel flow in a flume with smooth bed and glass side walls. A number of numerical tests were carried out to select an appropriate value of the dispersion coefficient. The value was selected by fitting simulated curves to the rising limbs of concentration-time profiles. The resulting dispersion coefficient, 0.002m$^2$ sec$^{-1}$ for both flow and slow zones produced reasonably comparable profiles to measured data. The dimensionless values of the dispersion coefficients ($0.002/h_n U_g$) were also found to be near to the non-dimensional dispersion coefficient 0.0048/3 proposed by Chikwendu and Ojiakor (1985).

Figs. 9 and 10 show experimental and numerical concentration-time profiles from Series 1 and 2 (Set 1). As shown in these figures, the agreement between measured and numerical concentration profiles improved in the streamwise direction. This is probably due to the idealized initial conditions impose on the mathematical model. The simulated concentration profiles
exhibited long tails, consistent with the measured data. In Series 1 the experimental and numerical peak concentration decayed as a function of $t^{0.33}$ and $t^{0.80}$ between 2m and 4m from the tracer injection, respectively. The experimental and numerical peak concentration decayed as a function of $t^{0.39}$ and $t^{0.53}$ between 8m and 10m from the tracer injection. The peak concentration difference and time to peak difference between measured and numerical values at 10m were 2.5% and 3.5%, respectively.

Figs. 11 and 12 show the mean travel time and temporal variance of measured and simulated data. A step-like change in mean travel time and temporal variance shows the effect of pools and riffles on the longitudinal dispersion process. The simulated curves showed a linear increase in mean travel time and temporal variance, which were consistent with the measured data. In this dataset, the effect of smooth riffles on longitudinal dispersion was negligible.

In Set 2, the depth of pools was reduced from 60mm to 35mm keeping other dimensions constant. The hydraulic and other input parameters to the numerical solutions are given in Tabs. 1 and 2. Figs. 13 and 14 show experimental and numerical concentration-time profiles of Series 3 and 4 (Set 2). The skewed distributions and long tails of the simulated concentration profiles are consistent with the measurements. The peak concentration difference and time to peak difference between measured and numerical values at 10m from the tracer injection were 6.21% and 1.25%, respectively (Series 3, Set 2). The experimental and numerical peak concentration decay rate reduced along the flume in the streamwise direction.

Figs. 15 and 16 show mean travel time and temporal variance of measured and simulated data. The profiles show a linear increase in mean travel time and temporal variance with different slopes on pools and riffles. The numerical results are in agreement with the observed values. However, the values of mean travel time and temporal variance were lower than Set 1. This was due to a decrease in the slow zone area of the pools and increase in average velocity of slow zones.

Tab. 3 presents mean cloud velocity ($U_{\text{cloud}}$) and overall longitudinal dispersion coefficient ($K_x$) of measured and simulated concentration-time data of Series 1 and 2 (Sets 1) and Series 3 and 4 (Set 2). The measured and simulated $U_{\text{cloud}}$ showed variance of 2.2% and 5.47% in Series 1 and 2 (Set 1). The longitudinal dispersion coefficient ($K_x$) also showed differences of 9.14% and 11.31% in Series 1 and 2 (Set 1). Similar variations in mean cloud velocity and longitudinal dispersion coefficients were noted for other data set. The
longitudinal dispersion coefficient \( (K_x) \) increased with the increase in pool depth and discharge.

Fig. 9. Experimental and numerical concentration-time profiles.
Obr. 9. Experimentální a numerický průběh závislosti koncentrace na case; Concentration – koncentrace, Time – cas.

Fig. 10. Experimental and numerical concentration-time profiles.
Obr. 10. Experimentální a numerický průběh závislosti koncentrace na case.
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Fig. 11. Experimental and numerical mean travel time. Series 1 and 2 (Set 1).

Obr. 11. Experimentální a numerický prubeh závislosti střední doby postupu na vzdálenost. Série 1 a 2 (sada 1); Mean travel time – střední doba postupu, Distance – vzdálenost.

Fig. 12. Experimental and numerical temporal variance. Series 1 and 2 (Set 1).

Obr. 12. Experimentální a numerický prubeh závislosti variace střední doby postupu na vzdálenost. Série 1 a 2 (sada 1); Temporal variance – časová variace, Distance – vzdálenost.
Fig. 13. Experimental and numerical concentration-time profiles.
Obr. 13. Experimentální a numerický průběh závislosti koncentrace na čase; Concentration — koncentrace, Time — čas.

Fig. 14. Experimental and numerical concentration-time profiles.
Obr. 14. Experimentální a numerický průběh závislosti koncentrace na čase; Concentration — koncentrace, Time — čas.
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Fig. 15. Experimental and numerical mean travel time. Series 3 and 4 (Set 2).
Obr. 15. Experimentální a numerický průbeh závislosti střední doby postupu na vzdálenosti. Série 3 a 4 (sada 2); Mean travel time – střední doba postupu, Distance – vzdálenost.

Fig. 16. Experimental and numerical temporal variance. Series 3 and 4 (Set 2).
Obr. 16. Experimentální a numerický průbeh závislosti variace střední doby postupu na vzdálenosti. Série 3 a 4 (sada 2); Temporal variance – casová variace, Distance – vzdálenost.
Table 1. Hydraulic parameters used in Sets 1 and 2 of the numerical solution.

<table>
<thead>
<tr>
<th>Set 1</th>
<th>Series 1</th>
<th>Q [m$^3$s$^{-1}$]</th>
<th>Flow depth in pool Z1 [m]</th>
<th>Flow depth on riffle Z2 [m]</th>
<th>Velocity in pool Z1 [m s$^{-1}$]</th>
<th>Velocity on riffle Z2 [m s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.002</td>
<td>0.023</td>
<td>0.06</td>
<td>0.22</td>
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<td>2</td>
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<td>0.028</td>
<td>0.06</td>
<td>0.24</td>
<td>0.024</td>
</tr>
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<td>3</td>
<td>3</td>
<td>0.004</td>
<td>0.032</td>
<td>0.035</td>
<td>0.31</td>
<td>0.082</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.005</td>
<td>0.035</td>
<td>0.035</td>
<td>0.35</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 2. The dispersion and mass exchange coefficients employed in numerical solutions.

<table>
<thead>
<tr>
<th>Set 1</th>
<th>Series 1</th>
<th>Q [m$^3$s$^{-1}$]</th>
<th>m [m$^2$sec$^{-1}$]</th>
<th>$k^x m/wU^x$</th>
<th>D [m$^2$sec$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.002</td>
<td>0.0019</td>
<td>0.027</td>
<td>0.002</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.003</td>
<td>0.0031</td>
<td>0.0315</td>
<td>0.002</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.004</td>
<td>0.0026</td>
<td>0.029</td>
<td>0.002</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.005</td>
<td>0.0029</td>
<td>0.029</td>
<td>0.002</td>
</tr>
</tbody>
</table>

$U^*$ – average flow zone velocity of the pool; $U^*$ – prumerná rychlost proudu v tuni.

Table 3. Experimental and simulated values of mean velocity and longitudinal dispersion coefficients of concentration-time profiles.

<table>
<thead>
<tr>
<th>Set 1</th>
<th>Flow Rate$^{1)}$ (Series 1)</th>
<th>(Series 2)</th>
<th>(Series 3)</th>
<th>(Series 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{U}_{cloud}$ (exp) [m sec$^{-1}$]</td>
<td>0.001091</td>
<td>0.1460</td>
<td>0.2392</td>
<td>0.2866</td>
</tr>
<tr>
<td>$\bar{U}_{cloud}$ (simu) [m sec$^{-1}$]</td>
<td>0.1067</td>
<td>0.1380</td>
<td>0.2319</td>
<td>0.2743</td>
</tr>
<tr>
<td>$K_r$(exp) [m$^2$ sec$^{-1}$]</td>
<td>0.0350</td>
<td>0.0389</td>
<td>0.0283</td>
<td>0.0334</td>
</tr>
<tr>
<td>$K_r$(simu)[m$^2$ sec$^{-1}$]</td>
<td>0.0318</td>
<td>0.0433</td>
<td>0.0211</td>
<td>0.0264</td>
</tr>
</tbody>
</table>


6. Conclusions

A two-zone model has been proposed to simulate longitudinal transport in channels with pools and riffles. Both zones have similar advection-dispersion
equations linked by mass exchange terms. This two-zone model was solved using implicit finite-difference numerical schemes and the results were compared with known analytical solutions and the experimental data.

This study has demonstrated the effectiveness of the numerical schemes to solve the two-zone model. Further, the proposed two-zone model takes into account the advection and dispersion of the both zones and therefore, improves the understanding of longitudinal transport in channels of this type. The advection and dispersion terms of the slow zone help in simplifying the calculation of dead zone parameters with varying discharges. It is noted that the boundary between the two zones is not associated with any large change in the properties of the flows.

A systematic comparison of experimental and numerical results from geometries of idealized pools and riffles shows good agreement for the general shape, peak and time to peak of concentration profiles. The pools appear to be the main reason for the rapid decay of the peak concentration and the persistence of the long tails of the pollutant. Further work is proposed to develop this approach for field application.

Acknowledgment. The Ministry of Education, Govt. of Pakistan supported the second author for the duration of this study.

List of symbols

\( A_n \) – cross-sectional area of the flow in zone \( n \) \( [m^2] \),
\( A^{(i)}, P^{(i)} \) – tridiagonal matrices,
\( B^{(i)} \) – diagonal matrix,
\( C_0 \) – cross-sectional average solute concentration,
\( C_n \) – cross-sectional average solute concentration in zone \( n \),
\( C_n(x,t) \) – concentration in zone \( n \) at longitudinal spatial co-ordinate \( x \) at time \( t \),
\( C_{n,j} \) – value of \( C_n \) at grid point \( x_j, t_k \),
\( C_{n,j}^k \) – vector of concentration values \( C_{n,j} \),
\( D_n \) – dispersion coefficient in zone \( n \) \( [m^2 \text{ sec}^{-1}] \),
\( h_n \) – depth of zone \( n \),
\( m \) – mass exchange coefficient between flow and slow zones per unit length per unit time \( [m^2 \text{ sec}^{-1}] \),
\( M' \) – total mass of solute \( [mg] \),
\( Q_n \) – flow rate in zone \( n \) \( [m^3 \text{ sec}^{-1}] \),
\( T(x) \) – mean travel time of tracer clouds \( [sec] \),
\( T^2(x) \) – temporal variance of tracer clouds \( [sec^2] \),
\( U_n \) – average flow velocity in zone \( n \) \( [m \text{ sec}^{-1}] \),
\[ U_f \] friction velocity \([\text{m sec}^{-1}]\),
\[ w \] channel width \([\text{m}]\),
\[ ?_n \] reciprocal of the normalized cross-sectional area of zone \( n \),
\[ ?(x) \] Dirac delta function,
\[ ?x \] mesh size used in finite difference scheme,
\[ ?t \] time step used in finite difference scheme,
\[ ?_? \] forward difference operator,
\[ ?_0 \] central difference operator,
\[ C \] concentration difference between flow and slow zones,
\[ ?(n) \] \(-?\)-th iteration approximation \( c_n^{(k)} \),
\[ ? \] normalized mass exchange rate coefficient,
\[ ?_n \] normalized cross-sectional area of zone \( n \).

REFERENCES

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APPENDIX. MASS EXCHANGE COEFFICIENTS

The laboratory experiments were performed to study mass exchange mechanism between flow and slow zones of a pool. Several writers [see for example Westrich, 1976; Valentine and Wood, 1977; Tsai and Holley (1979); Seo, 1990a; Elliott and Brooks, 1997a, 1997b] had undertaken experimental studies to investigate the mass exchange mechanism in the channels. In this study the mass exchange coefficients were measured at various stations along a typical pool of Geometry 1 (pool depth 0.06m) for a variety of flow rates. The longitudinal dimension of the pool was varied in a systematic way, using rectangular blocks of concrete keeping other dimensions same. The mass exchange coefficients were measured at various stations at the interface and at mid pool depth for a variety of flow rates.
To compute mass exchange coefficients between the flow and slow zones in the pool, the dispersion Equat. (1) of the slow zone was simplified by ignoring advection and dispersion terms. These assumptions were made to simplify the analysis of mass exchange coefficients, and explain why the resulting experimental values of these coefficients for higher pool length were greater than the values used in the numerical simulations. It was further assumed that the tracer solution was directly introduced in the slow zone and the concentration in the flow zone was considered to be zero. Denoting the slow zone concentration by \( S \), the dispersion equation was simplified as

\[
\frac{A_s \frac{\partial S}{\partial t}}{\partial t} = mS. \tag{A-1}
\]

On integration this gives

\[
m = \frac{(\ln S_0 - \ln S) A_s}{t}, \tag{A-2}
\]

where \( A_s \) is the area and \( S_0 \) is the concentration in the slow zone of the pool at \( t = 0 \). Equat. (A-2) indicates that \( m \) could have constant values if the concentration decayed exponentially.

A trend of increase in mass exchange coefficients with the increase in flow rate was observed, but a clear relation did not emerge from the results.
Moreover, the results showed the effects of change in longitudinal dimension of a pool on mass exchange coefficients. The pool eventually transformed into a dead zone of water by decreasing its length. The mass exchange coefficients decreased by reducing the pool length, which might be due to decreasing advection effects. Table A-1 present the runs average mass exchange coefficient computed at various stations for the Geometry 1-5 shown in Fig. A-1. Table A-2 presents a comparison of non-dimensional mass exchange coefficients with earlier studies.

![Diagram showing probe positions in the pool.](image)

**Fig. A-2.** A line diagram showing probe positions in the pool.

**Table A-1.** Average mass exchange coefficients in the pool as described by Geometry 1-5.

<table>
<thead>
<tr>
<th>Discharge [m³ sec⁻¹]</th>
<th>Mass exchange coefficient (m) [m² sec⁻¹]</th>
<th>K = (m/w) [m sec⁻¹]</th>
<th>k* = (k/U*</th>
<th>? = (A_S/m) [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.002</td>
<td>0.0023</td>
<td>0.0074</td>
<td>0.0367</td>
<td>8.08</td>
</tr>
<tr>
<td>0.003</td>
<td>0.0035</td>
<td>0.0113</td>
<td>0.0478</td>
<td>5.31</td>
</tr>
<tr>
<td>0.004</td>
<td>0.0038</td>
<td>0.0122</td>
<td>0.042</td>
<td>4.89</td>
</tr>
<tr>
<td>0.005</td>
<td>0.0042</td>
<td>0.0135</td>
<td>0.053</td>
<td>4.42</td>
</tr>
</tbody>
</table>

U* – average flow zone velocity of the pool/prumerná rychlost proudu v tuni, ? – tracer residence time in slow zone of the pool/rezidencní doba stopovace vpomalé zóne tune, k* – non-dimensional mass exchange coefficients/bezrozmerný soucinitel výmeny hmoty.
Table A-2. Comparison of non-dimensional mass exchange coefficients with earlier studies.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Type</th>
<th>$Ld$</th>
<th>$k^T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Westrich (1976)</td>
<td>Cube</td>
<td>0.2 –  4</td>
<td>0.013 ? 0.02</td>
</tr>
<tr>
<td>Valentine and Wood (1977)</td>
<td>Strips</td>
<td>1 ? 6.7</td>
<td>0.014 ? 0.026</td>
</tr>
<tr>
<td>Tsai and Holley (1979)</td>
<td>Cube</td>
<td>1 –  4</td>
<td>0.016 ? 0.021</td>
</tr>
<tr>
<td></td>
<td>Dune</td>
<td>9.6</td>
<td>0.027 ? 0.061</td>
</tr>
<tr>
<td>Tsai and Holley (1979)</td>
<td>Strip</td>
<td>8.0</td>
<td>0.083 ? 0.096</td>
</tr>
<tr>
<td>Seo (1990)</td>
<td>Pool</td>
<td>10.5 – 11.3</td>
<td>0.044 ? 0.071</td>
</tr>
<tr>
<td>Present study</td>
<td>Pool</td>
<td>4.66 – 23.3</td>
<td>0.0367 ? 0.060</td>
</tr>
</tbody>
</table>

DVOUZÓNOVÝ MODEL PODÉLNÉ DISPERZE V KORYTECH S IDEALIZOVANÝMI TUNEMI A PRAHY

Eric M. Valentine, Zulfiqar Ali a David C. Swailes


Proto je pro popis podélné disperze v korytech se soustavou opakujících se tuní a prahu navržen jednorozmerný dvouzónový matematický model, obsahující dvojici rovnic pro advektivní disperzi doplněných výrazem pro prenos hmoty.

Je použita numerická metoda konečných diferencí a její vhodnost je ověřována porovnáním se známým analytickým rešením. Nadto byla provedena rada pokusů s podobnou disperzi pro ručně jednoduché geometrie soustavy tuní a prahu v laboratorním žlabu. Pro kalibraci dvouzónového modelu byly porovnány výsledky s odpovídajícími analytickými rešením.

Tato studie prokázala vhodnost numerických metod pro rešení dvouzónového modelu. Navržený dvouzónový model uvažuje advekci a disperzi obou zón, a tedy zlepšuje porozumení mechanismu podélného transportu v korytech tohoto typu. Systematická srovnávání experimentálních a numerických výsledků pro koryto složené z idealizovaných tuní a prahu ukázala dobrou shodu jak obecněho tvaru koncentracního profilu, tak doby dosažení maxima koncentrace.
A two-zone model for longitudinal dispersion in channels with idealized pools and riffles

Seznam symbolů

- $A_n$ – prutocná plocha v zóně $n$ [m$^2$],
- $A^{0}, P^{0}, B^{0}$ – matice,
- $C_0, C_n$ – prumerná koncentrace roztoku v prurezu, v zóně $n$,
- $C_n(x,t)$ – koncentrace v zóně $n$ (v místě souradnice $x$ a v case $t$),
- $C_{n,i}^k$ – hodnota $C_n$ v bode hřízky $x = x_i, t = t_k$,
- $C_{n,j}^k$ – vektor koncentrace $C_{n,j}^k$,
- $D_n$ – součinitel disperse v zóně $n$ [m$^2$ s$^{-1}$],
- $h_n$ – hloubka v zóně $n$,
- $m$ – součinitel výmeny hmoty mezi prutocnou a pomalou zónou za jednotku délky a casu,
- $M'$ – celková hmota rozpuštené látky [mg],
- $Q_n$ – prutok v zóně $n$ [m$^2$ s$^{-1}$],
- $T(x)$ – střední doba postupu stopovacího mraku [s],
- $T^2(x)$ – casová variace stopovacího mraku [s$^2$],
- $U_n$ – prumerná rychlost proudu v zóně $n$ [m s$^{-1}$],
- $U^{*}$ – trecí rychlost [m s$^{-1}$],
- $W$ – šírka kanálu [m],
- $\gamma_n^+$ – reciprocní hodnota normalizované prurezové plochy v zóně $n$,
- $d(x)$ – Diracova delta funkce,
- $dx$ – velikost síte ve schématu konečných diferencí,
- $dt$ – casový krok ve schématu konečných diferencí,
- $\gamma^*$ – dopredná diference,
- $\gamma^0$ – centrální diference,
- $\gamma C_n$ – rozdíl koncentrace mezi prutocnou a pomalou zónou,
- $\gamma C_n(\tau)$ – aproximace $C_n^k$ v $\tau$-té iteraci,
- $e$ – normalizovaný součinitel výmeny hmoty,
- $\gamma_n$ – normalizovaná prurezová plocha v zóně $n$. 