Stackelberg Assumption vs. Nash Assumption in Partially Cooperative Games

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Abstract We deal with $n$-person normal form games where a subset of players decide to cooperate (signatories) and choose strategies by maximizing the aggregate welfare of the coalition members as in International Environmental Agreements (IEA) context. The non-cooperating (non-signatories) players choose their strategies as a Nash equilibrium. In this paper the partial cooperative equilibrium (PCE) under the Nash-Cournot and the Stackelberg assumptions are considered and presented also in the case of non-signatories multiple decision. Some properties are discussed in both situations, particularly the profit of the players are compared.

Keywords Partial cooperation, Stackelberg assumption, Nash-Cournot assumption, Stackelberg leader’s value

JEL classification C72, C79

1. Introduction

In the recent years different authors have investigated in a game theory context situations where some of the agents behave non-cooperatively (non-cooperating players or non-signatories), the rest of them sign an agreement (cooperating players or signatories). This mixture of cooperation and noncooperation developed in the International Environmental Agreements (IEA) context, recall for instance the Helsinki and Oslo Protocols on the reduction of sulphur signed in 1985 and 1994, and the Kyoto Protocol on the reduction of greenhouse gases causing global warming signed in 1997 (see Finus 2001). In these situations, usually, only a portion of the involved countries sign an agreement: this leads to coalition formation processes and to partial cooperative equilibrium concepts (see, for example, Barrett 1994; Beaudry et al. 2000; Carraro and Siniscalco 1992; Chakrabarti et al. 2011; Mallozzi and Tijs 2008a, 2008b, 2009; Ray and Vohra 1997; Yi 1997).

A three-stage game describes the problem of IEA: in the first stage (coalition formation game) players decide whether to participate in an agreement, in the second stage (partial cooperative game) they choose the emission levels and in the third one (cooperative game) the welfare obtained among the coalition members is allocated according to a sharing rule.

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We are interested in the second stage that corresponds to a partial cooperative situation where the signatories as well as the non-signatories have to decide the strategy. This can be formalized in two ways: all the players decide simultaneously (Nash-Cournot assumption, see Carraro and Siniscalco 1992) or the cooperating members are assumed to be leaders in the process and the players decide sequentially (Stackelberg assumption, see Barrett 1994). In this last case it is assumed that the non-signatories choose a Nash equilibrium profile.

The concept of partial cooperative equilibria (PCE) has been presented in the symmetric case, i.e. for identical players, and for quadratic payoff functions in Barrett (1994) and in Carraro and Siniscalco (1992). In Mallozzi and Tijs (2008a, 2008b) the problem has been investigated in a general framework and the existence of PCE under the Stackelberg assumption has been proved for potential games and also for aggregative games. In Mallozzi and Tijs (2008b) the case of multiple equilibria for the non-signatories has been considered in the symmetric case and, by using a selection in the set of the equilibria, a definition of PCE has been given.

In this paper we present the definition of PCE in the general case of multiple non-signatories decision by assuming the signatories to be risk-adverse as well as to be optimistic to force the non-signatories decision. Moreover, in line with previous papers dealing with Cournot duopolies (see Amir and Grilo 1999; Okuguchi 1999; Sheraly et al. 1983) or fishing conflicts (see Denisova and Garnaev 2008), we compare the Nash-Cournot and the Stackelberg one: the question is when the aggregate value of the cooperating players, i.e. the total amount they receive, under the Nash-Cournot assumption is lower or greater than the one they receive under the Stackelberg assumption. Some considerations for the non-signatories are done. Section 2 contains the models formulation and Section 3 contains the discussions about the signatories and non-signatories payoffs.

2. Partially cooperative models

Let \( \Gamma = \langle n; X_1, \ldots, X_n; f_1, \ldots, f_n \rangle \) be an \( n \)-person normal form game with player set \( I = \{1, 2, \ldots, n\} \), \( n \in \mathbb{N} \), with strategy space \( X_i \) and profit function \( f_i : X \rightarrow \mathbb{R} \) for player \( i \in I \), being \( X = X_1 \times \ldots \times X_n \). We denote by \( x_{i,j} \) the vector \( (x_i, x_{i+1}, \ldots, x_{j-1}, x_j) \) and by \( X_{i,j} \) the cartesian product \( X_i \times X_{i+1} \times \ldots \times X_{j-1} \times X_j \) for any \( i, j \in \{1, \ldots, n\} \) and \( i < j \). Here we do not precise any assumption on the strategy sets \( X_1, \ldots, X_n \) as well as on the profit functions \( f_1, \ldots, f_n \). We assume the existence of a PCE profile in order to compare different models (for existence results, see Chakrabarti et al. 2011; Mallozzi and Tijs 2008a, 2008b).

As in IEA context, we suppose now that a fixed group of the \( n \) players participate in an agreement, say \( P_{k+1}, \ldots, P_n \), the rest of the players \( P_1, \ldots, P_k \) acting in a noncooperative way for each \( k = 0, \ldots, n \) (we mean by \( k = 0 \) that all the players participate to an agreement, by \( k = n \) that no player participates to an agreement). In this case \( k \) is called the level of non-cooperation and it is assumed to be given. Cooperating players or signatories choose strategies by maximizing the aggregate welfare of the coalition.
members, i.e.

\[ F_k(x_1, \ldots, x_n) = \sum_{j=k+1}^{n} f_j(x_1, \ldots, x_n). \]  

(1)

The non-signatories play as singletons and choose their strategies as a Nash equilibrium with payoffs \(f_1, \ldots, f_k\). For \(k = 0\) all the players are signatories and maximize their joint payoff \(\sum_j f_j(x_1, \ldots, x_n)\) ending up to a social optimum equilibrium; for \(k = n\) all the players are non-signatories and we have a Nash equilibrium problem for all \(n\) players.

There are mainly two assumptions regarding the sequence of moves in the above scheme: (i) the Nash-Cournot assumption if all the players choose their strategies simultaneously (see Carraro and Siniscalco 1992); (ii) the Stackelberg assumption if the players choose their strategies sequentially (see Barrett 1994).

2.1 Nash-Cournot assumption

Each player chooses the strategy taking into account the optimality of the other players as in the Nash equilibrium concept. Given the level of non-cooperation \(k\), the signatories choose their strategy \((y_{k+1}, y_{k+2}, \ldots, y_n) = y_{k+1,n} \in X_{k+1,n}\) and the first \(k\) players with payoffs \(f_i, i = 1, \ldots, k\), do not participate to the agreement and play as singletons, all the players deciding together. More precisely, we look for a vector \(x^{\text{NC}}(k) = (\bar{x}_1, \ldots, \bar{x}_k, \bar{x}_{k+1,n}) \in X\) such that for any \(i = 1, \ldots, k\)

\[ f_i(\bar{x}_1, \ldots, \bar{x}_n) = \max_{y \in X_i} f_i(\bar{x}_1, \ldots, \bar{x}_{i-1}, y, \bar{x}_{i+1}, \ldots, \bar{x}_k, \bar{x}_{k+1,n}) \]

and also

\[ F_k(\bar{x}_1, \ldots, \bar{x}_n) = \max_{y_{k+1,n} \in X_{k+1,n}} F_k(\bar{x}_1, \ldots, \bar{x}_k, y_{k+1,n}) \]

\[ = \max_{y_{k+1,n} \in X_{k+1,n}} \sum_{j=k+1}^{n} f_j(\bar{x}_1, \ldots, \bar{x}_k, y_{k+1,n}) \]

where \(F_k\) is defined in (1).

**Definition 1.** A vector \(x^{\text{NC}}(k) = (\bar{x}_1, \ldots, \bar{x}_k, \bar{x}_{k+1,n}) \in X\) satisfying the above Nash equilibrium requirements is called a partial cooperative equilibrium under the Nash-Cournot assumption of the game \(\Gamma\) where \(n-k\) players sign the agreement. The value \(F_k(x^{\text{NC}}(k))\) is called the aggregate welfare of the signatories under the Nash-Cournot assumption and level of non-cooperation \(k\).

The definition of partial cooperative equilibrium under the Nash-Cournot assumption has been introduced by Carraro and Siniscalco (1992) for symmetric players in the quadratic case, then studied by Beaudry et al. (2000) and Chakrabarti et al. (2011) in a more general context.
2.2 Stackelberg assumption

In this case the game is a two-stage game: we assume the Stackelberg leadership of the signatories which act as a single player and announce their joint strategy. In fact, in IEA context it may be argued that signatories are better informed than non-signatories about emission levels in other countries since they coordinate their environmental policies within an IEA (see Finus 2001, ch. 13). Non-signatories are the followers and react by playing a non-cooperative game: they choose a Nash equilibrium in a $k$-person subgame. The solution is given then by using backward induction.

More precisely, given the level of non-cooperation $k$, the signatories choose their strategy $(y_{k+1}, y_{k+2}, \ldots, y_n) = y_{k+1,n} \in X_{k+1,n}$ and the first $k$ players with payoffs $f_i(x_1, \ldots, x_k, y_{k+1,n})$ for any $i = 1, \ldots, k$ do not participate to the agreement and choose a Nash equilibrium against the strategy $y_{k+1,n}$.

Denote by $\Gamma_k(y_{k+1,n}) = \langle k; X_1, \ldots, X_n; f_1, \ldots, f_k \rangle$ the $k$-person game with strategy spaces $X_1, \ldots, X_k$ and payoff function $f_i(x_1, \ldots, x_k, y_{k+1,n})$ of player $i$, $i = 1, \ldots, k$, and by $NE_k(y_{k+1,n})$ the set of the Nash equilibrium profiles. By $NE_k$ we mean the correspondence mapping any $y_{k+1,n} \in X_{k+1,n}$ into the set $NE_k(y_{k+1,n}) \in X_{1,k}$. If the game $\Gamma_k(y_{k+1,n})$ has a unique Nash equilibrium $(\eta_1(y_{k+1,n}), \ldots, \eta_k(y_{k+1,n}))$ for any $y_{k+1,n}$, the signatories $P_k+1, \ldots, P_n$ maximize the aggregate welfare function defined in (1) and solve the problem

$$\max_{y_{k+1,n} \in X_{k+1,n}} F_k(\eta_1(y_{k+1,n}), \ldots, \eta_k(y_{k+1,n}), y_{k+1,n}).$$

**Definition 2.** A vector $x^{ST}(k) = (\eta_1(\xi_{k+1,n}), \ldots, \eta_k(\xi_{k+1,n}), \xi_{k+1,n}) \in X$ such that $\xi_{k+1,n}$ solves the problem (2) is called a partial cooperative equilibrium under the Stackelberg assumption of the game $\Gamma$ where $n-k$ players sign the agreement. The value $F_k(x^{ST}(k))$ is called the aggregate welfare of the signatories under the Stackelberg assumption and level of non-cooperation $k$.

The definition of partial cooperative equilibrium under the Stackelberg assumption has been introduced by Barret (1994) for symmetric players in the quadratic case, then studied by Mallozzi and Tijs (2008a) for symmetric potential games having a unique symmetric Nash equilibrium for non-signatories, then for aggregative games (see Mallozzi and Tijs 2008b) in a general context.

In the trivial coalition case, i.e. when only one player behaves as signatory, Definition 1 gives the Nash equilibrium solution and Definition 2 gives the Stackelberg-Nash equilibrium solution with one leader (the only signatory) and $k$ followers playing a Nash game (see Sheraly et al. 1983).

It is well known in the one leader case that the leader’s profit is higher under the Stackelberg assumption than the Nash-Cournot one, as proved in Başar and Olsder (1995, Proposition 3.16 for cost functions) and in Sheraly et al. (1983, Lemma 6 for an oligopolistic market with a leader firm). It is easy to see that we have the same result with $n-k$ cooperating leaders according to the Definitions 1 and 2.

**Proposition 1.** If the game $\Gamma_k(y_{k+1,n})$ has a unique Nash equilibrium for any $y_{k+1,n} \in X_{k+1,n}$, then

$$F_k(x^{ST}(k)) \geq F_k(x^{NC}(k)).$$
The same result is given also in Finus (2001, Proposition 10.1) for identical signatories having quadratic profit functions in IEA framework. The proof of the Proposition 1 is not discussed here: a more general result (see Proposition 2) will be proved in Section 3.

**Example 1.** Let us consider $n = 4$, $X = [0, 1]^4$ and the following payoffs:

\[
\begin{align*}
    f_1(x_1, x_2, x_3, x_4) &= x_1 - x_4 x_2^2 \\
    f_2(x_1, x_2, x_3, x_4) &= -(x_1 - x_2 - x_3)^2 \\
    f_3(x_1, x_2, x_3, x_4) &= x_2 - x_3^2 x_3 \\
    f_4(x_1, x_2, x_3, x_4) &= x_2 (x_3 - x_1)
\end{align*}
\]

If we consider $k = 2$ signatories, player 3 and 4, their aggregate payoff is $F_2(x_1, x_2, x_3, x_4) = x_2 - x_2^2 x_3 + x_2 (x_3 - x_1)$. Under the Nash-Cournot assumption there is one partial cooperative equilibrium $x^{NC}(2) = (1, 0, 1, 0)$ with aggregate welfare $F_2(x^{NC}(2)) = 0$. Under the Stackelberg assumption, for any $(x_3, x_4) \in X$ there is a unique Nash equilibrium for players 1 and 2, namely $NE_2(x_3, x_4) = \{(1, 1 - x_3)\}$ and the signatories problem is

\[
\max_{(x_3, x_4) \in X^2} F_2(1, 1 - x_3, x_3, x_4).
\]

The partial cooperative equilibrium in this case is $x^{ST}(2) = (1, 1/2, 1/2, 0)$ with aggregate welfare $F_2(x^{ST}(2)) = 1/4 > F_2(x^{NC}(2))$.

### 3. Non-uniqueness of signatories decisions

As discussed in Mallozzi and Tijs (2008b, Example 1), the uniqueness assumption of the Nash equilibrium $(\eta_1(y_{k+1,n}), \ldots, \eta_k(y_{k+1,n}))$ of the game $\Gamma_k(y_{k+1,n})$ not always occurs: a public goods game with quadratic technology constraints shows the non-signatories multiple decisions. Then, as done in one leader case (see Başar et al. 1995; Breton et al. 1988), one may assume a different behavior of the leading coalition.

Suppose now that we have a correspondence mapping to any $y_{k+1,n} \in X_{k+1,n}$ the set of the Nash equilibrium profiles of the $k$ non-signatories, i.e. $NE_k(y_{k+1,n}) \subset X_{1,k}$. If the coalition of the signatory players has an optimistic view about the non-signatory choice in the set $NE_k(y_{k+1,n})$, then the coalition members maximize the function

\[
G_k^*(y_{k+1,n}) = \max_{x_{1,k} \in NE_k(y_{k+1,n})} \sum_{j=k+1}^n f_j(x_{1,k}, y_{k+1,n}).
\]

(3)

On the other hand, if the coalition has a pessimistic behavior, the coalition members maximize the worst can happen when the non-signatories choose in the set $NE_k(y_{k+1,n})$, namely the function

\[
G_k^w(y_{k+1,n}) = \min_{x_{1,k} \in NE_k(y_{k+1,n})} \sum_{j=k+1}^n f_j(x_{1,k}, y_{k+1,n}).
\]

(4)
Definition 3. A vector $x^s(k) = (\xi_{1,k}, \xi_{k+1,n}) \in X$ such that
\[
\xi_{k+1,n} \in \operatorname{argmax}\{G^s_k(y_{k+1,n}), y_{k+1,n} \in X_{k+1,n}\}
\]
and $\xi_{1,k} \in NE_k(\xi_{k+1,n})$ is called a strong partial cooperative equilibrium of $\Gamma$; a vector $x^w(k) = (\xi_{1,k}, \xi_{k+1,n}) \in X$ such that
\[
\xi_{k+1,n} \in \operatorname{argmax}\{G^w_k(y_{k+1,n}), y_{k+1,n} \in X_{k+1,n}\}
\]
and $\xi_{1,k} \in NE_k(\xi_{k+1,n})$ is called a weak partial cooperative equilibrium of $\Gamma$.

In the context of hierarchical two-stage games with one leader the above definition corresponds to the concept of strong hierarchical Nash equilibrium and weak hierarchical Nash equilibrium, widely studied in the literature.

The values $G^s_k(\xi_{k+1,n})$ and $G^w_k(\xi_{k+1,n})$ are called the strong and the weak aggregate welfare of the signatories under the Stackelberg assumption and level of non-cooperation $k$, respectively.

We have that $G^w_k(\xi_{k+1,n}) \leq G^s_k(\xi_{k+1,n})$ and the equality holds if the game $\Gamma_k(y_{k+1,n})$ has a unique Nash equilibrium $(\eta_1(y_{k+1,n}), \ldots, \eta_k(y_{k+1,n}))$ for any $y_{k+1,n}$.

It is possible to prove that the strong aggregate welfare of the signatories under the Stackelberg assumption is higher than the aggregate welfare under the Nash-Cournot assumption, in the sense of the following Proposition.

Proposition 2. Let $x^{NC}(k) = (\bar{x}_{1,k}, \bar{x}_{k+1,n}) \in X$ be a partial cooperative equilibrium under the Nash-Cournot assumption and $x^s(k) = (\xi_{1,k}, \xi_{k+1,n}) \in X$ be a strong partial cooperative equilibrium under the Stackelberg assumption. Then
\[
G^s_k(\xi_{k+1,n}) \geq F_k(x^{NC}(k)).
\]

Proof. By contradiction, let be $G^s_k(\xi_{k+1,n}) < F_k(x^{NC}(k)) = F_k(\bar{x}_{1,k}, \bar{x}_{k+1,n})$, i.e.
\[
\max_{y_{k+1,n} \in X_{k+1,n}} \max_{x_{1,k} \in NE_k(y_{k+1,n})} \sum_{j=k+1}^n f_j(x_{1,k}, y_{k+1,n}) < \sum_{j=k+1}^n f_j(\bar{x}_{1,k}, \bar{x}_{k+1,n}).
\]

Then, for any $y_{k+1,n} \in X_{k+1,n}$ we have
\[
\max_{x_{1,k} \in NE_k(y_{k+1,n})} \sum_{j=k+1}^n f_j(x_{1,k}, y_{k+1,n}) < \sum_{j=k+1}^n f_j(\bar{x}_{1,k}, \bar{x}_{k+1,n}).
\]

By choosing $y_{k+1,n} = \bar{x}_{k+1,n}$
\[
\max_{x_{1,k} \in NE_k(\bar{x}_{k+1,n})} \sum_{j=k+1}^n f_j(x_{1,k}, \bar{x}_{k+1,n}) < \sum_{j=k+1}^n f_j(\bar{x}_{1,k}, \bar{x}_{k+1,n}),
\]

we have a contradiction since $\bar{x}_{1,k} \in NE_k(\bar{x}_{k+1,n})$. □

If there is a unique Nash equilibrium for the non-signatories, the result in Proposition 2 is nothing but the result in Proposition 1. Let us note that the inequality can be strict as in the following example.
Example 2. Let us consider $n = 2, k = 1, X = [0, 1]^2$ and the following payoffs:

$$f_1(x_1, x_2) = (1/2 - x_2)x_1$$
$$f_2(x_1, x_2) = x_1x_2$$

Under the Nash-Cournot assumption the partial cooperative equilibria (Nash equilibria) are the pairs $(x_1, x_2) \in X$ such that $x_1 = 0, x_2 \in [1/2, 1]$ with aggregate welfare $F_1(x^{NC}(1)) = 0$. Under the Stackelberg assumption, for any $x_2 \in [0, 1]$ the function in (3) is given by $G^1_2(x_2) = x_2$ if $x_2 \leq 1/2$ and 0 otherwise, so the strong partial cooperative equilibria are the pairs $(x_1, x_2) \in X$ such that $x_1 \in [0, 1], x_2 = 1/2$ with aggregate welfare $G^1_2(1/2) = 1/2 > F_1(x^{NC}(1))$.

The inequality proved in Proposition 2 may be not true anymore if one considers a weak partial cooperative equilibrium as in the following example.

Example 3. Let us consider $n = 2, k = 1, X = \{a, b\}^2$ and the following payoffs

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0,0</td>
<td>-1,1</td>
</tr>
<tr>
<td>b</td>
<td>-2,-1</td>
<td>-1,-2</td>
</tr>
</tbody>
</table>

Under the Nash-Cournot assumption $x^{NC}(1) = (a, b)$ is the partial cooperative equilibrium with aggregate welfare $F_1(x^{NC}(1)) = 1$. Under the Stackelberg assumption the function in (4) is given by $G^w_1(a) = 0, G^w_2(b) = -2$, so the weak partial cooperative equilibrium is $(a, a)$ with aggregate welfare $G^w_1(a) = 0 < F_1(x^{NC}(1))$.

Non-signatories aggregate welfare. Let us point out that from the non-signatories point of view, things go differently with respect to the signatories. For a given level of non-cooperation $k$, let us define the aggregate welfare of the non-signatories, i.e.

$$H_k(x_1, \ldots, x_n) = \sum_{j=1}^{k} f_j(x_1, \ldots, x_n).$$

Let $x^{NC}(k) \in X$ and $x^{ST}(k) \in X$ be a partial cooperative equilibrium under the Nash-Cournot and the Stackelberg assumption respectively; in case of multiple non-signatories Nash equilibria let $x^*(k) \in X$ and $x^w(k) \in X$ be a strong and a weak partial cooperative equilibrium under the Stackelberg assumption. The values $H_k(x^{NC}(k))$ and $H_k(x^{ST}(k))$ are called the aggregate welfare of the non-signatories under the Nash-Cournot and the Stackelberg assumption, respectively; analogously we define $H_k(x^*(k))$ and $H_k(x^w(k))$ and call them the strong and the weak aggregate welfare of the non-signatories, respectively.

In Example 1 we have that $H_2(x^{NC}(k)) = H_2(x^{ST}(k)) = 1$ and also in Example 2 we have $H_1(x^{NC}(k)) = H_1(x^*(k)) = 0$, so for the non-signatories it is equivalent to act under the Nash-Cournot and the Stackelberg assumption. In Example 3, the situation for the non-signatories is better under the Stackelberg assumption with respect to the
Nash-Cournot one. In fact, in this case we have \( H_1(x^{NC}(k)) = -1 < H_1(x^{w}(k)) = 0 \).

A classical example, as the Cournot oligopoly, shows that it can be also worst for the non-signatories to assume the Stackelberg assumption.

**Example 4.** Let us consider a 2-firm competitive market of a single homogeneous commodity with linear inverse demand function \( P(Q) = \alpha - Q \). Any firm \( i \) can supply \( q_i \in X = [0, q^0] \) \( (q^0 > 0) \) with cost \( c \) per unit produced \( (\alpha > c \geq 0) \). If \( Q = q_i + q_{-i} \) is the aggregate quantity, firm \( i \) maximizes its profit

\[
\Pi_i(q_1, q_2) = q_i(\alpha - (q_i + q_{-i})) - cq_i.
\]

Let \( k = 1 \): the partial cooperative equilibrium (Cournot equilibrium) is \( x^{NC}(1) = \left( \frac{1}{3}(\alpha - c), \frac{1}{3}(\alpha - c) \right) \) with aggregate welfare for the non-signatory \( H_1(x^{NC}(1)) = \frac{1}{9}(\alpha - c)^2 \). If we assume that firm 2 is the signatory (leader in the Stackelberg setting) we have \( x^{ST}(1) = \left( \frac{1}{4}(\alpha - c), \frac{1}{2}(\alpha - c) \right) \) and the welfare for firm 1 is \( H_1(x^{ST}(1)) = \frac{1}{16}(\alpha - c)^2 < H_1(x^{NC}(1)) = \frac{1}{9}(\alpha - c)^2 \).

### 4. Conclusion

This paper deals with a partial cooperative game which is often encountered in the International Environmental Agreements (IEA) context, where a group of players plays cooperatively (i.e. acts as a single player) and the remaining players play among themselves according to the Nash equilibrium concept. We studied the relationship between this cooperating group (the signatories) and the other players (the non-signatories): either in Stackelberg or in Nash sense.

We distinguished unique and non-unique reactions of the non-signatories and in the latter case, the signatories can play optimistically (assuming that the reactions are most favorable) or pessimistically (assuming that the non-signatories choose the worst case within the optimal reaction set). Several properties in both assumptions have been presented and the welfare received by the players under the Stackelberg and the Nash assumptions are compared.

In line with the results proved in Sheraly et al. (1983), it would be interesting to compare the strategy chosen by the leading coalition in the Stackelberg and in the Nash case, not only the aggregate welfare. This turns out to be very important in the context of the global emission games and, in general, of IEA (see Finus 2001). This will be the object of future research.

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**References**


