Comparison of daily potential evapotranspiration calculated by two procedures based on Penman-Monteith type equation

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Abstract. Paper presents comparison of the daily reference crop (grass vegetation cover) potential evapotranspiration results calculated by the two modifications of the Penman-Monteith type equation. The first modification was published in FAO recommendation (Allen et al., 1998), PM-FAO, the second is modification according to Budagovskiy (1964) and Novák (1995), PM-BN. Both are used in soil water simulation models HYDRUS-1D and GLOBAL. Calculations were performed for frost-free seasons of the years 2000–2009, using the meteorological station Gabčíkovo (South Slovakia) meteorological data and canopy characteristics. The results indicate significant differences in daily and seasonal potential evapotranspiration. Reasons for those differences are discussed; they should be in different net radiation and aerodynamic resistance estimation methods.

Keywords: Potential evapotranspiration; Reference crop; Penman-Monteith equation.

INTRODUCTION

Evapotranspiration is the key element of land water balance structure and its precise evaluation is decisive for its proper evaluation. Annual evapotranspiration of Slovak territory covers approximately two thirds of precipitation, in lowland regions it approaches 90 percent of the annual precipitation (Majerčák and Šťastný, 2001).

Penman-Monteith (PM) equation and its modifications (Budagovskiy, 1964; Budagovskiy and Novák, 2011a, b; Monteith, 1965; Novák, 1989; Penman, 1948) are basic tools to calculate evapotranspiration. They are used mostly to calculate potential evapotranspiration and then to modify it to the actual one. There were indicated significant differences in results of mathematical modeling of water and energy transport in the Soil-Plant-Atmosphere System (SPAS), because the two mentioned different procedures based on modifications of the PM equation were used. One (Allen et al., 1998) is incorporated in the model HYDRUS-1D (Šimůnek et al., 2008), and the second in the model GLOBAL (Majerčák and Novák, 1992; Novák and Majerčák, 1992).

It was necessary to compare both approaches and find the reasons for the resulting differences. The best way how to do it, is to compare results of potential evapotranspiration of a reference crop (grass vegetation cover), calculated by both of the methods. This is the aim of this paper.

FAO Penman-Monteith equation (PM-FAO modification procedure)

Food and Agricultural Organization (FAO) procedure to calculate potential evapotranspiration \( ET_a \) is based on modification of Penman-Monteith equation in general form as it is implemented in the simulation model HYDRUS-1D (Šimůnek et al., 2008). It can be written:

\[
ET_a = ET_r + ET_a = \frac{1}{\lambda} \left[ \Delta (R_n - G) + \rho_a c_p (e_a - e_d)/r_a \right] + \frac{\rho_a c_p (e_a - e_d)/r_a}{\Delta + \gamma \left[ 1 + (r_c/r_a) \right]} \tag{1}
\]

where \( ET_r \) is the potential evapotranspiration rate expressed by a soil water layer thickness evaporated per day (mm d\(^{-1}\)), \( ET_a \) is „the radiation term” (mm d\(^{-1}\)), \( ET_r \) is „the aerodynamic term” (mm d\(^{-1}\)), \( \Delta \) is the slope of the saturation vapor pressure curve as a function of temperature \( (e_a = f(T)) \) (kPa °C\(^{-1}\)), \( R_n \) is net radiation (MJ m\(^{-2}\)d\(^{-1}\)), \( G \) is the soil heat flux (MJ m\(^{-2}\)d\(^{-1}\)), \( \rho_a \) is the air density (kg m\(^{-3}\)), \( c_p \) is the specific heat of moist air (kJ kg\(^{-1}\) °C\(^{-1}\)), \( c_p = 1.013 \) kJ kg\(^{-1}\) °C\(^{-1}\)), \( (e_a - e_d) \) is the vapor pressure deficit (kPa), \( e_a \) is the actual water vapor pressure (kPa), \( e_d \) is the actual water vapor pressure (kPa), \( r_a \) is the aerodynamic resistance (s m\(^{-1}\)), \( \lambda \) is the latent heat of vaporization (MJ kg\(^{-1}\)), \( \gamma \) is the psychrometric constant (kPa °C\(^{-1}\)), \( r_c \) is the canopy resistance (s m\(^{-1}\)).

Sensitivity analysis has shown net radiation \( R_n \) as the most sensitive parameter in the Eq. (1). Evaporating surface is strongly influencing evapotranspiration through the two resistances \( r_a \) and \( r_c \). Inversion term of resistances (used in some modifications of the Eq. (1)), is the velocity coefficient of the turbulent transport \( D = 1/r_a \) (m s\(^{-1}\)).

Aerodynamic resistance \( r_a \) is expressed in the FAO procedure (Allen et al., 1998), as follows:

\[
r_a = \frac{\ln \left( \frac{z_m - d_z}{z_{om}} \right) \ln \left( \frac{z_h - d_z}{z_{oh}} \right)}{k^2 u_z}, \tag{2}
\]

where \( z_m \), \( z_h \) are heights of the wind speed and of the temperature and humidity measurements (m), \( d_z \) is the zero plane displacement height (m), \( z_{om} \) is the roughness length governing transport of the heat and the vapor (m), \( z_{om} \) is the roughness length governing momentum transfer (m), \( u_z \) is the wind speed at the height \( z \) (m s\(^{-1}\)), \( k \) is the von Karman constant (-) (\( k = 0.41 \)).

According to the FAO procedure of potential evapotranspiration calculation – \( ET_a \) is a reference evapotranspiration, calculated by the PM-FAO equation (Eq. (1)), (Allen et al., 1998). It is potential evapotranspiration of „reference” grass canopy with height \( z_p = 0.12 \) m, canopy resistance \( r_c = 70 \) s m\(^{-1}\), and albedo \( a = 0.23 \).
Aerodynamic resistance of reference canopy is expressed by Allen et al. (1998):

\[ r_u = \frac{208}{u_2^2}, \quad (3) \]

where \( u_2 \) is the wind speed at the height 2 m.

Crop canopy resistance is (Allen et al., 1998):

\[ r_c = \frac{r_l}{0.5LAI} = \frac{200}{LAI}, \quad (4) \]

where \( r_l \) is the daily average bulk stomata resistance of the well illuminated leaf (s m\(^{-1}\)) \((r_l \approx 100 \text{ s m}^{-1}\)), LAI is the active leaf area index.

Then, Penman-Monteith equation (Eq. (1)) with the above mentioned data for grass reference surface is (Allen et al., 1998):

\[
ET_o = ET_r + ET_a = 0.408(\frac{R_n - G}{\Delta + \gamma(1 + 0.34\mu_2)}) + \frac{900}{\Delta + \gamma(1 + 0.34\mu_2)} \binom{\gamma}{T + 273} e_a - e_d \binom{\gamma}{2}.
\]

\[ \Delta = \frac{q_o - q}{\Delta + \gamma(1 + 0.34\mu_2)}. \quad (5) \]

**Penman-Monteith equation and modified calculation procedure (PM-BN)**

The modification of Penman-Monteith equation and calculation procedure was re-developed using the same principle and equations like before by Penman (1948), but using newer information about boundary layer of the atmosphere (BLA) by Monin and Obukhov (1954). Continuous modifications were also published by Budagovsky (1964), Novák (1989), Novák and Hurtalová (1987). The principle of „the big leaf“ allows to calculate potential evapotranspiration based on the horizontal wet surface with zero canopy resistance \( r_l = 0 \). Eq. (6) was used to calculate \( ET_o \) in simulation models GLOBAL and model HYDRUS-ET (note – it differs from HYDRUS-1D) (Novák, 1995; Majerčák and Novák, 1992; Novák and Majerčák, 1992; Šimůnek et al., 1997):

\[ ET_o = ET_r + ET_a = \frac{\phi(R - G)}{c_p + L\phi} + \frac{\rho a c_p D d'}{c_p + L\phi}. \quad (6) \]

where \( ET_o \) is the potential evapotranspiration intensity (kg m\(^{-2}\) s\(^{-1}\)), \( R \) is net radiation at the evaporation surface level (W m\(^{-2}\)), \( G \) is the soil heat flux (W m\(^{-2}\)), \( \rho_a \) is the air density (kg m\(^{-3}\)), \( c_p \) is the moist air specific heat (J kg\(^{-1}\) K\(^{-1}\)) \((c_p = 1005 \text{ J kg}^{-1} \text{ K}^{-1})\), \( D \) is the velocity coefficient of turbulent transport (m s\(^{-1}\)), \( d' \) is the air saturation deficit, expressed as the difference of the specific humidity \( d' = q_o - q, q_o \) and \( q \) are specific air humidity of air saturated by water vapor and measured air humidity (kg kg\(^{-1}\)), \( L \) is the latent heat of vaporization (J kg\(^{-1}\)) \((L = 2.46 \times 10^6 \text{ J kg}^{-1} \text{ at } T = 20\text{°C})\), \( \phi \) is the slope of vapor pressure curve.

The air saturation deficit \( d' = q_o - q \) can be replaced by the saturation deficit \( d = e_o - e_s \) using partial pressures \( d' = 6.22 \times 10^{-4} (d_0 - q, T \) is the air temperature at 2-m height.

New knowledge of the BLA physics allowed to develop equation for the turbulent transport \( D \) calculation (Budagovsky, 1964; Monin and Obukhov, 1954; Novák and Hurtalová, 1987). For neutral state of the atmosphere, equation for \( D = 1/r_u \) can be written (Novák, 1995):

\[ D = \frac{ku_a}{v_a} \left( \frac{z_o}{v_a} \right)^{0.5} + \ln \left( \frac{z_o - d}{z_o} \right), \quad (7) \]

where \( u_a \) is the friction velocity (m s\(^{-1}\)), \( z_o \) is the roughness length of the evaporating surface (m), \( v_a \) is the air kinematic viscosity (m s\(^{-1}\)).

**RESULTS AND DISCUSSION**

To compare quantitatively the results of reference canopy potential evapotranspiration \( ET_o \) performed by two modifications of Penman-Monteith equations (PM-FAO, PM-BN) the daily rates were calculated for the frost-free period from the 2\(^{nd}\) of March to the 8\(^{th}\) of December (later noted as a season) with standard meteorological characteristics and data from meteorological station Gabčíkovo for seasons of the 2000–2009 period.

\( ET_o \) seasonal totals of 10 years (2000–2009) calculated by the two described methods are presented in Fig. 1. Significant differences in results can be seen; the average seasonal \( ET_o \) difference is 217 mm of the water layer; the relative difference is 0.36. Minimum difference \( ET_o \) was calculated for year 2004 (24%) – 193 mm of the water layer – and maximum difference was in the year 2009 (31%) – 238 mm of the water layer.

![Fig. 1. Daily cumulative values ETo, calculated according to the PM-FAO and PM-BN for the seasons of years 2000–2009. (Gabčíkovo, South Slovakia.)](image-url)
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radiation as well as aerodynamic terms of both equations are expressed in a different way. Radiation term of the PM-FAO method is indirectly proportional to the wind velocity. In the PM-BN modification it does not depend upon the wind velocity.

The reasons of such significant differences in daily totals of \( E_{T_o} \) calculated by the two methods can be attributed to several facts. Primarily to differences in the net radiation calculation methods. In principle, the both methods use the same method for the net radiation calculation \( R_n \), but there are differences in long-wave radiation balance calculation \( R_{nl} \). The PM-FAO method does not account for differences between the surface evaporation body temperature \( T_s \). As the temperature of the evaporating (radiating) surface the standard air temperature \( T \) at reference level is used, i.e. at the 2 m elevation. It is assumed that the both temperatures are the same. This will lead to the lower calculated net long-wave radiation \( R_{nl} \) and to higher values of net radiation \( R_r \). Modified PM-BN method accounts for the temperature difference \( T_s - T \) on \( R_{nl} \) by an iteration process (Budagovskiy, 1964; Novák, 1987, 1995). Recently, Widmoser (2009) proposed iterative method to calculate surface temperature to increase the PM method accuracy too.

Aerodynamic resistances calculated by the Eqs (3) and (7) as a function of wind velocity \( u \) are in Fig. 3. The difference between results calculated by the same net radiation \( (to evaluate the sensitivity of both models to the aerodynamic term) was 12% (season of the year 2000) and 5% (season 2009). \( E_{T_o} \) values calculated by the PM-FAO method were always higher.

Both methods were verified in a different way. The PM-FAO method was verified for different climate conditions on lysimeters data (Allen et al., 1989; Allen et al., 1998).

The PM-BN method was verified comparing calculated daily evapotranspiration rates with those, estimated and measured by
the energy balance method at experimental field with maize canopy at Trnava (South Slovakia) by Novák and Hurtalová (1987). The same type of verification was performed within the NOPEX Project field campaign at Uppsala, Sweden for grass, spring barley, rice and winter wheat (Hurtalová et al., 1996; Novák et al., 1997).

Which method can be recommended? This question could be answered only, if verification of both methods would be implemented with the same models (HYDRUS and GLOBAL) data input sets, and the results compared with evapotranspiration real observation (determined by e.g. lysimeters), not by some other calculations. So far such verification has not been performed. This could be the subject of the future project within the Central European regional cooperation.

CONCLUSIONS

Results of potential evapotranspiration of reference grass canopy calculated by the two modifications of the Penman-Monteith equation application showed significant differences.

Higher values of $E_T$ were calculated by the PM-FAO method (Allen et al., 1998) for Gabčíkovo meteorological station, (average seasonal value 818 mm for seasons 2000–2009). Significantly lower (602 mm) values were calculated by the PM-BN method according to Novák (1989).

The reasons of these differences can be found in different methods of long-wave radiation evaluation and the aerodynamic resistance calculation. Long-wave radiation calculation method used in the PM-FAO method does not involve the evaporating (radiating) surface temperature of the radiating surface, but air temperature at standard height. Aerodynamic resistance is neglecting the resistance of the surface sublayer ($0, z_s$).

The aim of this paper was not to prefer one method to the other, but to compare two methods of potential evapotranspiration calculation. In view of wide use of the both methods and also of the both mentioned numerical models, we tried to stress the possible results differences in their respective use. Also to point out the sources of such sometimes quite serious differences. Users should be aware of these at their use for scientific purposes, as well as, for practical applications (e.g. for irrigation water need forecasts).

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