The Discrete Charm of Uniform Linear Pricing of an Input Production Joint Venture

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Abstract A popular way of obtaining essential inputs is based on the establishment of an input production joint venture (IPJV) in the upstream (U) section of the vertical chain of production by firms competing and selling final goods downstream (D). Different governances may be designed for the management of an IPJV according to the ownership structure, the degree of delegation granted to the IPJV by parent firms and the extent of competition in the D market. Industry optimal arrangements with nonlinear pricing may be hard to implement and may be banned by regulators, mainly in the case of minimal delegation based on coordination (collusion) among the D firms. A handy and endurable governance turns out to be maximal delegation, i.e., an independent IPJV, seasoned with linear uniform pricing, even if this solution may contain some inefficiency.

Keywords Input production joint venture, nonlinear pricing, product differentiation, delegation

JEL classification L24, L42

1. Introduction

A popular way of obtaining intermediate products is based on the establishment of an input production joint venture (IPJV) in the upstream (U) section of the vertical chain of production by firms competing in the downstream (D) market. Examples may be found in almost all industries producing both manufactured goods and services (Chen and Ross 2003; Hewitt 2008; Höffler and Kranz 2011; Rossini and Vergari 2011). For instance, Fiat and GM buy diesel engines from a jointly owned dedicated firm called VM Motors. With this vertical arrangement the two D firms jointly set up and own an enterprise specialized in the production of an input (diesel engine) exclusively sold to themselves for the production of automotive vehicles.2 Several governances of the IPJV may be conceived and distinguished according to the degree of delegation and/or freedom granted by parent firms, to the ownership structure, to the pricing policies, to

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2 Most IPJV are confined to exclusive dealings, while a small percentage allows for the sale of the jointly manufactured input to alien firms (see Hewitt 2008). An IPJV usually sells intermediate goods to its owners, competing in D. Thanks to competition in D exclusive dealings of the IPJV with parent firms may be tolerated by regulatory authorities. In this paper we restrict the analysis to the most common IPJV. Notice that in our simple duopoly scheme both firms are served.
the degree of heterogeneity of D firms joining the IPJV and, finally, to the extent of competition in the D market. In this variety of settings implementation problems occur due to the complexity of vertical contracts which pushes firms to move quite often to simple, yet suboptimal, linear pricing.

This paper wants to explore preferred IPJV governances with linear and nonlinear pricing of the input. The IPJV framework is the distinctive feature of this paper that let us depart from past and existing literature on affine vertical issues (see among others Katz 1987; De Graba 1990; Yoshida 2000; Inderst and Shaffer 2011). We shall see that in the IPJV some settings turn out to be hard to accept mostly because of their apparent collusive character. When D firms are heterogeneous due to different costs the immediate consequence may be input price discrimination by the IPJV established in U. In such a case, if the input can be exchanged, arbitrage may take place among D firms. Price discrimination may, after all, be quite unpalatable between two partners who have jointly given birth to a new company, the IPJV, but in some cases remains a privately efficient device. Last but not the least, a regulator, or simply a civil court, may intervene and ban discrimination on the basis of common law criteria. By investigating the area of unworkable and/or (practically and legally) unsustainable vertical arrangements we find that uniform linear pricing with a vertically independent IPJV is the handiest and most reasonable set up.

In a simple duopoly with linear and nonlinear pricing we shall analyze the general case of an asymmetric framework, i.e., heterogeneous firms in D, first, considering an IPJV totally independent of the owners (maximal delegation), secondly, by examining the opposite case when the D firms dictate the IPJV the maximization of aggregate vertical profits (minimal delegation). The latter arrangement, where the D firms state the IPJV policy, carries a problem. It is a privately efficient solution for the industry, yet it mimics a vertical cartel since it is based on an explicit apparent coordination (collusion) of D firms on the input prices which may not be legally feasible.

At the end of the day, the most practical and simple setting, which provides the escape route from the pitfalls of many vertical arrangements, requires maximal delegation, i.e., independence of the IPJV in the U section, coupled to a uniform linear input price for the D firms. This solution has also some private and social advantage over linear price discrimination. Our conclusion adds a further explanation of why independent IPJVs and linear pricing are so common in all industries, especially in manufacturing, even though they are suboptimal. Last but not the least, maximal delegation may be even more desirable as it emerges, in many cases, as the aftermath of a legal functional unbundling requirement, i.e., mandatory functional vertical separa-

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3 These are the most relevant vertical arrangements among a large existing variety, comprising for instance retail price maintenance (RPM) and bilateral monopoly (see Inderst 2010).
4 Hewitt (2008) and Nakamura (2005) provide rich empirical evidence about joint ventures settings.
5 For linear vs. nonlinear contracts see the recent surveys by Inderst (2010) and Miklós-Thal et al. (2010).
For applications of linear pricing see Inderst and Shaffer (2009) and Arya et al. (2008). An empirical test of linear vs. non linear pricing may be found in Bonnet et al. (2006).
6 That can be seen, in a different framework with product homogeneity and no vertical common ownership, in Yoshida (2000).
7 This solution closely parallels one of the most common escape ways followed in cases of deadlocks in decision making of joint ventures (Hewitt 2008, p. 234).
ration, set by regulators towards an existing vertically integrated entity (Höffler and Kranz 2011). Our results add to existing literature since they come from a framework where an IPJV operates in a scenario of product market differentiation.

The paper is organized as follows. In Section 2 we outline the model. Then, we proceed with nonlinear pricing in Section 3 and, subsequently, we go through linear pricing in Section 4. In Section 5 we draw our conclusions.

2. The model

We figure out an industry made up of two manufacturers of a final differentiated good competing in a Cournot mode. We assume horizontal product differentiation. The differentiated output, \( q_i \), is sold at the unit price \( p_i \), while variable production costs are \( c_i q_i \) with \( i = 1, 2 \). Firm 1 is less efficient than firm 2, namely \( c_1 - c_2 \geq 0 \), with \( c_2 = 0 \). Thus, the parameter \( c_1 \in [0, 1] \) measures the extent of heterogeneity among the two firms, while \( c_1 = 0 \) represents the symmetric scenario. Linear demand schedules are \( p_i = a - q_i - bq_j \) in the region of quantities at positive prices with \( i \neq j \). Parameter \( a > 0 \) represents market size; \( b \in [0, 1] \) measures substitutability between final goods and/or the degree of competition in the D market. The essential input required for the manufacturing of the final good is produced by an Input Production Joint Venture (IPJV) which is a limited liability Equity Joint Venture (Hewitt 2008, chap. 5) between the two downstream (D) companies which jointly own the IPJV. Profits of the IPJV accrue to the D parent firms making for their consolidated profits.\(^8\) As for the D technology we assume that one unit of input is embodied in each unit of output (perfect vertical complementarity). Input production takes place at zero cost, for the sake of simplicity.

We conduct the analysis assuming that, before the formation of the IPJV, there is a U monopolist input producer and two D firms. This situation represents also D firms’ outside option. The IPJV may be the result either of the joint establishment of a brand new productive entity after the shut down of the incumbent U monopolist or it may come from the joint acquisition of the U monopolist by the two D firms. Or, finally, it may be the aftermath of an ownership and/or functional unbundling imposed by the regulator on a previously vertically integrated firm. In all cases we consider an IPJV company in U which is a jointly owned corporate, i.e., a limited liability corporate with an independent identity (Hewitt 2008, p. 59–60). This means that, if the U capital stock is, for instance, assumed to be equal to the fixed cost, set to zero for the sake of simplicity, losses drive the company to bankruptcy and shut down. We adopt the same legal organizational structure for the D companies. The objective function of the IPJV is:

\[
\pi_{IPJV} = d\pi_U + (1 - d) \sum_{i=1,2} \pi_{C_i},
\]

which is a convex combination of consolidated profits of the D parents (\( \pi_{C_i} \)) and profits

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\(^8\) Alternative vertical arrangements to obtain the essential input are analysed in Inderst (2010), Inderst and Shaffer (2009) and Rossini and Vergari (2011).
raised in $U$ ($\pi_U$).$^9$ $d \in [0, 1]$ is the delegation parameter, i.e., the incentive structure for the managers governing the IPJV. The degree of delegation is an inverse measure of the weight the D firms have in the executive board of the IPJV. We go through the choice of the governance of the IPJV focusing on the two extremes of maximal and minimal delegation. Under maximal delegation, that may be the result of an unbundling ruling from a regulating court, $d = 1$, the IPJV is autonomous as the D firms do not interfere at all in the input price decision, i.e., the IPJV maximizes its own profit as a wholly autonomous entity, without caring about what happens to parents D. At the other extreme of minimal delegation, $d = 0$, the IPJV complies with the D guidelines, that is the D firms decide not to delegate the input price decision.$^{10}$

In these two polar cases we examine the interaction between the D firms and the IPJV as a three stage game solved by backward induction. The first stage of the game regards the D firms’s joint choice of delegation; the second stage touches upon the selection of the pricing contract for the input; the third stage describes Cournot-Nash quantity competition among the D firms. In the first stage, containing the joint decision of the two D firms as to delegation, we confine to symmetric solutions whereby the two firms decide in a coordinated way to grant either maximal or minimal delegation. We are not concerned with off diagonal solutions where one firm adopts maximal while the rival/partner opts for minimal. We exclude these asymmetric outcomes since they are not consistent with the two firms setting up a joint venture which is supposed to emerge as the result of an agreement containing also the degree of delegation granted to the IPJV.

We conduct our investigation under the assumption that the more efficient firm is not able to throw the rival out of the market. Formally:

**Assumption 1.** $c_1 \leq a (1 - b)$.

In what follows, we consider in turn nonlinear and linear pricing for the provision of the input by the IPJV to D parent firms. These are the most common arrangement contracts in vertical arm’s length relationships.

### 3. Nonlinear pricing

Nonlinear pricing is a policy adopted mostly by firms selling services and access to infrastructure facilities. It is quite common in regulated industries where it takes the form of a two-part tariff, composed by an access, or subscription fee, and a fixed price for each unit of the good or service delivered.

Formally, a two-part tariff contract $(w_i, f_i)$ for $i = 1, 2$ is based on $w_i \geq 0$, the per-unit input price, and $f_i \geq 0$, the access fee. Under non linear input pricing, the operative profit in the $U$ section of the vertical chain is as follows:

$$\pi_U = w_1 q_1 + w_2 q_2 + f_1 + f_2.$$  

$^9$ $\pi_U$ and $\pi_U$ will be defined later according to the scenario analysed.

$^{10}$ If $d \in (0, 1)$ the IPJV objective lies between the two extreme cases and the input price decision is shared between $U$ and the D firms.
Also we define the \textit{consolidated} profits of the two D firms as:

\[ \pi_{C_i} = (p_i - c_i - w_i) q_i - f_i + s_i \pi_U, \]  

(3)

where \( s_i \in (0, 1) \) is the share of the IPJV profits going to firm \( i \). Without loss of generality we set \( s_1 = s \) and, consequently, \( s_2 = 1 - s \). \textit{Consolidated} profits have two components. The first, \( \pi_{Oi} = (p_i - c_i - w_i) q_i - f_i \) is the own profit of each D firm that comprises only individual D profit.\(^{11}\) The second, \( s_i \pi_U \) is the share of U profit going to D parents. As for the value of \( s_i \), the profit shares of the firms giving rise to the IPJV, many contractual schemes are envisaged in the literature (Hewitt 2008, p. 188) and no general solution seems to be available.\(^{12}\)

As anticipated above, under maximal delegation, the input pricing decision is entirely left to the IPJV. Therefore, the objective of the IPJV is \( \pi_{IPJV} = w_1 q_1 + w_2 q_2 + f_1 + f_2 \) (formally substitute \( d = 1 \) in (1)). As for the D firms, to ensure that they do not interfere with the IPJV, when making their output decisions, their objective is their own profit \( \pi_{Oi} = (p_i - c_i - w_i) q_i - f_i \). Indeed, the alternative, i.e., the maximization of consolidated profits by the D firms is inconsistent with the definition of maximal delegation since it implies that D firms take care of IPJV profit eliminating IPJV autonomy.\(^{13}\) In contrast, under minimal delegation, the input pricing decision is taken by the D firms maximizing their consolidated profits (formally substitute \( d = 0 \) in (1)).

Under nonlinear pricing, we draw the following result:

\textbf{Proposition 1.} Assume that the IPJV is able to set a two-part tariff contract for the input prices. Granting maximal delegation to the IPJV (\( d = 1 \)) the D parent firms are able to implement the cartel outcome in the absence of any explicit coordination (collusion). The optimal contracts are the following (\( M \) stands for maximal delegation):

\[ w_1^M (b, c_1) = \frac{b(a - ab + bc_1)}{2(1 - b)(b + 1)}, \quad f_1^M (b, c_1) = \frac{(ab - a + c_1)^2}{4(b + 1)^2(b - 1)^2} \]  

(4)

\[ w_2^M (b, c_1) = \frac{b(a - ab - c_1)}{2(1 - b)(b + 1)}, \quad f_2^M (b, c_1) = \frac{(a - ab + bc_1)^2}{4(b + 1)^2(b - 1)^2}. \]  

(5)

\textbf{Proof.} See the Appendix.

Under maximal delegation the IPJV via the contracts (4) and (5) implements the cartel solution. Thus, industry profits are maximized and a degree of delegation equal to 1 is optimal for the firms as a degree of delegation different from maximal delegation cannot improve the result. More precisely, under \textit{maximal delegation}, the U firm is totally autonomous in setting the two-part tariff contracts. Note that Cournot competition

\(^{11}\) Since the D firms are limited liability corporate enterprises and fixed costs are set to zero, we shall not consider the possibility of losses.

\(^{12}\) See for instance Van Long and Soubeyran (1999) for asymmetric contributions to research joint ventures.

\(^{13}\) The very definition of \textit{maximal delegation} is a situation where the IPJV is autonomous, that is the D (owner) firms do not interfere at all in the input price decision. A sufficient condition for this to occur is that when making their output decisions the D firms maximize their own profits without caring about the U profit. We show this statement in the Appendix.
is solved by maximizing D firms’ own profits rather than consolidated profits. In the Appendix, where a full proof of Proposition 1 is detailed, we show that maximization of consolidated profits by the D firms is inconsistent with maximal delegation since it implies that D firms take care of U profit eliminating U autonomy.

Discussion. From the above Proposition we can conclude that maximal delegation is optimal for the industry. As we prove below, minimal delegation may give rise to the same level of industry profits, but requires that the D firms explicitly coordinate (collude) their pricing policies. With maximal delegation the industry profits will be the same without any need for overt coordination. Indeed, the main difference between the two polar cases of delegation regards the distribution of industry profits along the vertical chain between the IPJV and the D firms which are the owners of the IPJV.\(^{14}\) Given Assumption 1, the equilibrium contracts in (4) and (5) specify positive per-unit prices. So that the optimal input prices are larger than those made within a vertically integrated firm where inputs are transferred along the vertical chain at an internal price equal to marginal cost, that in this case is zero. Comparing the two contracts we find that:

\[
\begin{align*}
w^M_1 - w^M_2 &= \frac{c_1 b}{2(1-b)} > 0, \\
f^M_1 - f^M_2 &= \frac{(c_1 - 2a)c_1}{4(b+1)(1-b)} < 0.
\end{align*}
\]

Equilibrium variables are:

\[
\begin{align*}
q^M_1 &= \frac{a(1-b) - c_1}{2(b+1)(1-b)}, \\
q^M_2 &= \frac{a(1-b) + bc_1}{2(b+1)(1-b)}, \\
q^M_1 - q^M_2 &= \frac{c_1 - 2a}{2(b+1)} < 0, \\
\pi^M_{O1} &= 0, \\
\pi^M_{O2} &= 0, \\
\pi^M_U &= \frac{2abc_1 - 2ac_1 + 2a^2 - 2a^2b + c_1^2}{4(b+1)(1-b)} - f.
\end{align*}
\]

In this case, since \(\pi_{Oi} = 0\), consolidated profits are simply given by the share of each D in the IPJV surplus.\(^{15}\) The above results replicate those of a multiproduct monopolist selling two differentiated goods in the D market: the IPJV implements the cartel outcome. More precisely, the IPJV adopts a policy of price discrimination between the

\(^{14}\) In our highly stilized scenario this difference is fairly unconsequential. However, there are many circumstances where it may matter, such as the case in which the IPJV and the D owners operate in distinct countries with different taxation regimes.

\(^{15}\) As detailed in the Appendix, \(\pi_{Oi} = 0\) depends on the choice of the outside option in the maximization problem solved by the IPJV. Namely, a positive outside option, independent of the per-unit price, would not change the equilibrium variables but it would change the distribution of profit along the vertical chain. Things change when the outside option depends on the optimal contract. See for instance Inderst and Shaffer (2011) that studies the optimal input contracts that an upstream monopolist is able to implement when it is constrained both by downstream competition and the threat of demand-side substitution.
two D firms. In particular the more efficient firm (firm 2) benefits from a lower per-unit price \( w_1^M > w_2^M \), but pays a larger fee. This result differs from part of previous literature cast in homogeneous linear pricing framework where no commonly owned U production facility such as the IPJV is considered (Katz 1987; De Graba 1990). At the equilibrium the competitive advantage is amplified and firm 2 efficiently produces a larger share of the total quantity. Inderst and Shaffer (2009) reach the same conclusion analyzing a different problem. They study optimal two-part tariff contracts in vertical relations with independent firms and show that a ban on price discrimination may reduce allocative efficiency and total welfare.

It is worth examining how the extent of price discrimination depends on firms’ asymmetry \( c_1 \) as well as on the degree of competition in the downstream market \( b \). The answer is contained in the following corollary.

**Corollary 1.**

(i) The extent of price discrimination goes up as firms’ cost asymmetry increases.

(ii) As competition in the D market becomes tougher, price divergence increases only if the efficiency gap is sufficiently high.

**Proof.**

(i) The following derivatives with respect to \( c_1 \) show that the per-unit input prices as well as the fees react in opposite directions to the increase in the efficiency gap:

\[
\frac{\partial w_1^M}{\partial c_1} > 0, \frac{\partial w_2^M}{\partial c_1} < 0, \frac{\partial f_1^M}{\partial c_1} < 0, \frac{\partial f_2^M}{\partial c_1} > 0.
\]

As the efficiency gap goes up, the IPJV shifts the quantity produced from firm 1 to firm 2. However, the most efficient firm is punished with a higher fee.

(ii) As for the effect of \( b \) on input prices, we have:

\[
\frac{\partial w_1^M}{\partial b} > 0, \frac{\partial w_2^M}{\partial b} > 0 \iff c_1 < a \frac{(b - 1)^2}{b^2 + 1} < a (1 - b).
\]

As competition becomes tougher the per-unit price of the less efficient firm always increases. The fiercer is competition in D, the higher will be the incentive to “shift” profit to U because the profit reservoir role played by the IPJV becomes more valuable. However, the effect of \( b \) on the per-unit price of the most efficient firm is twofold: for a low efficiency gap it increases with \( b \) (as the input price of the less efficient firm); if the efficiency gap is sufficiently large price discrimination widens.\(^{16}\) Finally, the fees vary with \( b \) in the following way:

\[
\frac{\partial f_1^M}{\partial b} < 0, \frac{\partial f_2^M}{\partial b} < 0 \iff c_1 < a \frac{(b - 1)^2}{b^2 + 1}.
\]

\(^{16}\) As a function of \( b \) the sign of \( \frac{\partial w_2^M}{\partial b} \) is: \( \frac{\partial w_2^M}{\partial b} > 0 \iff b < b_1 = \frac{a - \sqrt{c_1 (2a - c_1)}}{a - c_1} \in (0, 1) \), with \( \frac{\partial b_1}{\partial c_1} < 0. \)
Discussion. The outcome of the above Corollary departs from that obtained by part of previous literature in a linear pricing, homogeneous product framework without any IPJV in U (Katz 1987; De Graba 1990). Nonlinear pricing is not immune to price discrimination which gets wider as firms’ cost asymmetry increases and as competition in the D market becomes tougher, making for larger arbitrage incentives. These results raise some question marks on the implementation of nonlinear pricing in all cases where arbitrage activities may be carried out.\(^1\)

As for the other extreme of minimal delegation, the IPJV completely complies with D guidelines. Its objective function is simply the sum of the consolidated profits of Ds. Therefore in the second stage the Ds jointly set the optimal input contract by maximizing the objective
\[
\sum_{i=1,2} \pi_G^i; \quad while \quad in \quad the \quad third \quad stage \quad the \quad D \quad firms \quad independently \quad choose \quad their \quad quantities \quad competing \quad Cournot \quad style. \quad Proceeding \quad by \quad backward \quad induction, \quad we \quad find \quad that \quad the \quad optimal \quad fixed \quad fees (f_1 \quad and \quad f_2) \quad are \quad set \quad equal \quad to \quad zero, \quad (more \quad precisely \quad they \quad cancel \quad out \quad in \quad the \quad objective \quad function) \quad so \quad that \quad the \quad two-part \quad tariff \quad contracts \quad reduce \quad to \quad linear \quad contracts. \quad The \quad optimal \quad input \quad prices \quad are \quad then (m \quad stands \quad for \quad minimal \quad delegation):
\]
\[
w_1^m (b, c_1, s) = \frac{(a - ab + bc_1)b}{2 (s - 1) (b + 1) (b - 1)}, \quad (6)
\]
\[
w_2^m (b, c_1, s) = \frac{(a - ab - c_1)b}{2 (b + 1) (1 - b) s}. \quad (7)
\]

The two input prices are both positive under Assumption 1. Then, equilibrium quantities, prices, individual and industry profits under minimal delegation are:
\[
q_1^m = \frac{a - ab - c_1}{2 (b + 1) (1 - b)}, \quad q_2^m = \frac{a - ab + bc_1}{2 (b + 1) (1 - b)}
\]
\[
p_1^m = \frac{a + c_1}{2}, \quad p_2^m = \frac{a}{2}
\]
\[
\pi_1^m = \frac{(ab - a + c_1) (a - ab + bc_1)b}{4 (s - 1)(b + 1)^2 (b - 1)^2 s} - f
\]
\[
\pi_2^m = \frac{(a - ab - c_1) (a - c_1)}{4 (b + 1)(1 - b)}, \quad \pi_G^1 = \frac{(a - ab + bc_1)a}{4 (b + 1)(1 - b)}
\]
\[
\Pi^m = \frac{c_1^2 + 2abc_1 - 2ac_1 + 2a^2 - 2a^2 b}{4 (b + 1)(1 - b)} - f
\]

Looking at the equilibrium variables, we see that industry profits are the same as under maximal delegation, yet their distribution along the vertical chain, i.e., between the Ds and the IPJV, differs. As a matter of fact an IPJV where the D owner firms interfere with the input price decisions behaves like a multiproduct monopoly.\(^2\) Note that this result is quite predictable given that the input price decision is made maximizing industry profits. This may motivate the investigation of the Competition Authority that, in contrast, is not concerned a priori with maximal delegation since the decision is

\(^{17}\) Clearly, in the symmetric framework, that is when \(c_1 = 0\), price and fee discrimination disappears.

\(^{18}\) This result extends Chen and Ross (2003) to the case of asymmetric firms.
taken by the IPJV independently of the D firms. Minimal delegation is based on the explicit coordination of pricing policies by the D firms. Of course this is illegal and can be easily prosecuted. For this reason we exclude minimal delegation from the feasible governance in the case of nonlinear pricing. Notice that maximal delegation may be the aftermath of a legal unbundling requirement, i.e., mandatory vertical separation, set by regulators towards an existing vertically integrated entity (Höffler and Kranz 2011). In addition to that, nonlinear pricing is considered a vertical restraint and it is associated to price discrimination.

4. Linear pricing

In this section we investigate the case where the input price contract is a linear function of the total quantity purchased. Formally, under linear input pricing, the operative profits in the U section of the vertical chain are \( p_U = w_1q_1 + w_2q_2 \); and the consolidated profits of the two D firms are \( \pi_c = (p_i - c_i - w_i)q_i + s_i \pi_U \).

As shown in Section 3, with non linear pricing, the privately optimal governance of the IPJV requires maximal delegation. However, we have seen that this solution is not applicable in all circumstances. First, a two-part tariff may be subject to a ban by the regulator on price discrimination or firms can do arbitrage and thwart discrimination. Second, in service production and in most utilities “a linear price is the only two-part tariff that ensures universal service’’ (often imposed on regulated monopolies)... more generally optimal linear pricing is a good approximation to optimal two-part pricing when there is concern that a nonnegligible fixed premium would exclude either too many consumers (or firms)... or customers with low incomes” (or less efficient firms) (Laffont and Tirole 1993, p. 151). Third, two-part tariffs require complex contracts that are quite cumbersome to apply, in particular when the production costs of the D firms are not common knowledge. Fourth, as remarked by Villas-Boas (2007), two-part tariff contracts in the presence of uncertainty, have poor properties in terms of risk sharing. For all these reasons it seems worth investigating the optimal degree of delegation under linear pricing. As before, we consider and compare the two cases of maximal and minimal delegation.

4.1 Maximal delegation

Under maximal delegation, by definition, the IPJV maximizes its own profits and the D firms do the same. We distinguish between a price discrimination and a uniform price case. Under price discrimination the IPJV sets two input prices \((w_1, w_2)\); in the case of

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19 These are as expressions (2) and (3) with \( f_1 \) and \( f_2 \) equal to zero.
20 In many countries nondiscrimination laws are in effect. Consequently, it is forbidden to set different wholesale prices to providers of comparable services. Empirical studies usually assume a uniform linear price when estimating vertical contracts. See for instance Bonnet and Dubois (2010) and Bonnet et al. (2011) for two recent contributions.
21 “Linear pricing can be justified by the possibility of arbitrage” (Laffont and Tirole 1993, p. 151).
22 Italic words in brackets are ours.
A uniform input price the IPJV sets a unique input price \((w)\) for both D firms. We can then write the following:

**Proposition 2.** Assume that the IPJV is granted maximal delegation and that it is able to set a linear contract.

(i) **Under uniform linear pricing,** we have the following optimal contract:

\[
w^u(c_1) = \frac{2a - c_1}{4}.
\]  

(ii) **Under price discrimination,** we have the following optimal contracts:

\[
w_1^d(c_1) = \frac{a - c_1}{2},
\]

\[
w_2^d(c_1) = \frac{a}{2}.
\]

**Proof.** Solving backwards the vertical interaction, we find the following equilibrium variables. Under price discrimination (superscript \(d\) stands for price discrimination):

\[
w_1^d(c_1) = \frac{a - c_1}{2} \leq w_2^d(c_1) = \frac{a}{2},
\]

\[
p_1^d = \frac{a(b^2 + b - 6) + c_1(b^2 - 2)}{2(b^2 - 4)} \geq p_2^d = \frac{a(b^2 + b - 6) - bc_1}{2(b^2 - 4)},
\]

\[
q_2^d = \frac{2a - ab + bc_1}{-2(b^2 - 4)} \geq q_1^d = \frac{a(b - 2) + 2c_1}{2(b^2 - 4)},
\]

\[
q_2^d + q_1^d = \frac{2a - c_1}{2(2 + b)},
\]

\[
\pi_1^d = \frac{(a(b - 2) + 2c_1)^2}{4(b^2 - 4)^2} \leq \pi_2^d = \frac{(a(b - 2) - bc_1)^2}{4(b^2 - 4)^2},
\]

\[
\pi_U^d = \frac{a^2(b - 2) - a(b - 2)c_1 - c_1^2}{2(b^2 - 4)}.
\]

\[
\Pi^d = \pi_1^d + \pi_2^d + \pi_U^d = \frac{2a^2(b + 3)(b - 2)^2 + c_1^2(12 - b^2) - c_12a(b + 3)(b - 2)^2}{4(b + 2)^2(b - 2)^2}.
\]

Under uniform pricing (superscript \(u\) stands for uniform price):

\[
w^u(c_1) = \frac{2a - c_1}{4},
\]

\[
p_1^u = \frac{2a(b^2 + b - 6) + c_1(3b^2 + b - 6)}{4(b^2 - 4)} \geq p_2^u = \frac{2c_1 + (3 + b)(2a(b - 2) - bc_1)}{4(b^2 - 4)},
\]

\[
q_2^u = \frac{4a - 2ab + 2c_1 + 3bc_1}{-4(b^2 - 4)} \geq q_1^u = \frac{2a(b - 2) + (6 + b)c_1}{4(b^2 - 4)},
\]
\[ q_2^u + q_1^u = \frac{2a - c_1}{2(2 + b)}, \]
\[ \pi_1^u = \frac{(2a(b - 2) + (6 + b)c_1)^2}{16(b^2 - 4)^2} \leq \pi_2^u = \frac{(-2a(b - 2) + (2 + 3b)c_1)^2}{16(b^2 - 4)^2}, \]
\[ \pi_U^u = \frac{(c_1 - 2a)^2}{8(2 + b)}, \]
\[ \Pi^u = \frac{4a^2(b + 3)(b - 2)^2 + c_1^2(8b + 3b^2 + b^3 + 28) - 4c_1a(b + 3)(b - 2)^2}{8(b + 2)^2(b - 2)^2}. \]

Making the proper comparisons, we obtain that:

**Corollary 2.**

(i) The uniform price of the input is set between the two optimal input prices of price discrimination.

(ii) The profit of the less efficient D firm is always weakly lower than that of the most efficient firm. The sum of the profits of D firms is weakly larger with uniform pricing than with price discrimination. Industry profits are weakly larger with uniform pricing than with price discrimination. The IPJV profits are weakly smaller with uniform pricing than with price discrimination.

(iii) The consumer surplus and the social welfare are weakly larger in the case of a uniform input price.

**Proof.**

(i) As far as the optimal contracts are concerned we find that:
\[ w_i^d(c_1) \leq w_i^u(c_1) \leq w_i^d(c_1). \] (11)

(ii) As for the profits:
\[ \sum_{i=1}^{2} \pi_i^u - \sum_{i=1}^{2} \pi_i^d = \frac{3c_1^2}{8(b - 2)^2} \geq 0, \]
\[ \pi_U^u - \pi_U^d = \frac{c_1^2}{8(b - 2)} \leq 0, \]
\[ \Pi^u - \Pi^d = \frac{(1 + b)c_1^2}{8(b - 2)^2} \geq 0. \]

(iii) Finally, as for the prices, consumer surpluses and social welfare, we get:
\[ p_i^u \geq p_i^d \geq p_2^d \geq p_2^u, \] (12)
\[ CS^u - CS^d = \frac{3(1 - b)c_1^2}{16(b - 2)^2} \geq 0, \]
\[ SW^d - SW^u = \]
\[ = \frac{(2a^2(b-2)^2(7+3b)c_1+(28-5b^2)c_1^2)[4a(b-2)^2(7+3b)(c_1-a)+(b(b(9+b)-16)-76)c_1^2]}{128(b^2-4)^4} \leq 0. \]

**Discussion.** The above Corollary shows that *uniform linear pricing is privately and socially preferred with respect to input price discrimination*. This result is consistent with investigations cast in a homogeneous products without an IPJV commonly owned by D firms framework (Yoshida 2000). The intuition behind this result can be explained looking at the equilibrium prices (11) and (12). Indeed, under uniform pricing, the less efficient firm produces a lower quantity of output than under price discrimination so that overall the production is more efficiently divided among the two downstream firms. Clearly this difference in efficiency disappears as soon as \( c_1 = 0 \), i.e., firms costs converge.

### 4.2 Minimal delegation

Under minimal delegation, the D firms take care of the IPJV pricing policy. As in the non linear case, in the first stage the Ds jointly set the optimal input contract; while in the second stage the strategic interaction between the two D firms proceeds in their quantity competition. By definition, the IPJV is not functionally separated from the D owners. In this sense the setting is not one that complies with EU regulation on functional vertical unbundling. The equilibrium linear contracts coincide with the equilibrium contracts obtained under non linear pricing, expressions (6) and (7), where the optimal fixed fees were set equal to zero. We end up again with the monopoly outcome given that firms maximize their joint profits when setting the input prices.

**Proposition 3.** Under linear input pricing, the optimal (for the industry) degree of delegation that D parent firms grant to the IPJV is zero. Firms implement a cartel outcome and the optimal contracts are those displayed in expressions (6) and (7).

**Comment.** The above result, quite intuitive, implies a high risk for the D firms to be investigated by the Competition Authority for the pricing mechanism adopted as well as for the outcome that in both cases mimic a cartel. Granting maximal delegation to the IPJV may allow the D firms to escape from this risk: this is what we call “the discrete charm of (uniform) linear pricing.”

### 5. Conclusions

In our investigation of the IPJV governance we have come across four distinct settings. With nonlinear pricing maximal delegation to the U input producer may be optimal from the industry point of view, while minimal delegation may turn out legally unfeasible since it is based on explicit coordination (collusion) between D firms. However, also maximal delegation solution raises a feasibility issue due to price discrimination and to the fact that a two-part tariff is considered a vertical restraint, which may be
limited either by arbitrage or by the regulator. Moreover, the setting mimics a cartel outcome even if not based on overt coordination.

Linear pricing with maximal delegation emerges as a simple and viable recipe for an IPJV. An independently governed IPJV doing uniform linear pricing turns out to be the most likely, and quite often the unique, feasible arrangement. Moreover, uniform linear pricing has some welfare advantage over linear price discrimination, as already seen in Yoshida (2000). We have extended this result to product differentiation and to the joint ownership of U by D firms. Our result is also consistent with many stories of the IPJV, as the case in which the IPJV may be the aftermath of a legal unbundling requirement set by regulators towards an existing vertically integrated entity. As well explained by Höffler and Kranz (2011, p. 576): “Legal unbundling means that the essential input must be controlled by a legally independent entity with an autonomous management, but a firm which is active in the downstream market is still allowed to own this entity... but interferences in the entity’s operations are forbidden.” Legal unbundling has become quite common, especially in the EU, as a result of regulators’ guidelines. The joint ownership of a U firm released by a formerly vertically integrated firm may provide a reasonably acceptable solution, once U is made independent thanks to the IPJV and adopts uniform linear pricing.

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References


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Appendix

Proof of Proposition 1

Third stage quantity competition leads to the following quantities, prices and downstream own profits (as functions of the variables \(w_1\) and \(w_2\) and the parameter \(c_1\)).

\[
\begin{align*}
q_1 (w_1, w_2; c_1) &= \frac{2a - ab - 2c_1 - 2w_1 + bw_2}{(b + 2)(2 - b)}, \\
q_2 (w_1, w_2; c_1) &= \frac{2a - ab - 2w_2 + bc_1 + bw_1}{(b + 2)(2 - b)}, \\
p_1 (w_1, w_2; c_1) &= \frac{2a - ab + 2c_1 + 2w_1 + bw_2 - b^2c_1 - b^2w_1}{(b + 2)(2 - b)}, \\
p_2 (w_1, w_2; c_1) &= \frac{2a - ab + 2w_2 + bc_1 + bw_1 - b^2w_2}{(b + 2)(2 - b)}, \\
\pi_{01} (w_1, w_2; c_1) &= \frac{(2a - ab - 2c_1 - 2w_1 + bw_2)^2}{(b - 2)^2(b + 2)^2} - f_1, \\
\pi_{02} (w_1, w_2; c_1) &= \frac{(2a - ab - 2w_2 + bc_1 + bw_1)^2}{(b - 2)^2(b + 2)^2} - f_2.
\end{align*}
\]

In the second stage, the U firm faces the following maximization problem:

\[
\max_{w_1, w_2, f_1, f_2} \left[ w_1 q_1 (w_1, w_2; c_1) + w_2 q_2 (w_1, w_2; c_1) + f_1 + f_2 - f \right]
\]

s.t. \[
\frac{(2a - ab - 2c_1 - 2w_1 + bw_2)^2}{(b - 2)^2(b + 2)^2} - f_1 \geq 0, \\
\frac{(2a - ab - 2w_2 + bc_1 + bw_1)^2}{(b - 2)^2(b + 2)^2} - f_2 \geq 0, \\
w_1 \geq 0, w_2 \geq 0, f_1 \geq 0, f_2 \geq 0.
\]

As the first two constraints are binding in equilibrium, we have:

\[
\begin{align*}
f_1 (w_1, w_2; c_1) &= \frac{(2a - ab - 2c_1 - 2w_1 + bw_2)^2}{(b - 2)^2(b + 2)^2}, \\
f_2 (w_1, w_2; c_1) &= \frac{(2a - ab - 2w_2 + bc_1 + bw_1)^2}{(b - 2)^2(b + 2)^2}.
\end{align*}
\]

\(^{23}\) We solve the quantity competition stage assuming that the objective functions for the D firms are their own D profits. We discuss below the alternative of maximising their consolidated profits.

\(^{24}\) We are implicitly assuming that D firms’ outside option is zero. However, if the D firms do not form the IPJV we have a D duopoly with positive profits and an independent U monopoly. A change in the outside option does not modify the equilibrium output price and consolidated profit as it is independent of \(w_1\). It does modify the distribution of profit along the vertical chain, as it will be pointed out later.
The maximization problem, thus becomes:

$$\max_{w_1, w_2} \left[ w_1 q_1 (w_1, w_2; c_1) + w_2 q_2 (w_1, w_2; c_1) + f_1 (w_1, w_2; c_1) + f_2 (w_1, w_2; c_1) - f \right].$$

The solution implies the equilibrium contracts (4) and (5).

In the alternative scenario in which firms compete and maximize their consolidated profits with general share $s_1 = s_2 \in (0, 1)$, they compete also in $U$ which prevents the IPJV to behave in an independent way and which stops $U$ from being a profit reservoir. This can be clearly shown in the particular case in which $s = (w_1 q_1 + f_1)/(w_1 q_1 + w_2 q_2 + f_1 + f_2 - f)$, that is the $U$ surplus is allocated according to each firm’s contribution to the total revenue of $U$ which is proportional to profits. Formally, consolidated profits reduce to: $\pi_{C1} = (p_1 - c_1) q_1 - f_1$, $\pi_{C2} = p_2 q_2 - f_2$. Therefore, third stage competition implies standard duopoly quantities that are independent of $w_1$, $w_2$:

$$q_1 = \frac{a (2 - b) - 2 c_1}{(b + 2) (2 - b)},$$
$$q_2 = \frac{a (2 - b) + b c_1}{(b + 2) (2 - b)},$$
$$q_1 - q_2 = \frac{-c_1 (b + 2)}{(b + 2) (2 - b)} < 0.$$

In the second stage, the IPJV loses the control variables $w_1$ and $w_2$ and the optimal contract is $f_1 = \frac{(ab - 2a + 2c_1)^2}{(b+2)^2 (b-2)^2}$ and $f_2 = \frac{(2a - ab + bc_1)^2}{(b+2)^2 (b-2)^2}$ with $f_1 > f_2$. Remaining equilibrium variables are:

$$\pi_{O1}^* = 0, \quad \pi_{O2}^* = 0,$$
$$\pi_U^* = f_1 + f_2 - f = \frac{2a^2 (b - 2)^2 + c_1^2 (b^2 + 4) - 2c_1 a (b - 2)^2}{(b + 2)^2 (b - 2)^2} - f.$$

With this particular sharing rule firms behave as duopolists caring about their share of profits in $U$ and, therefore, competing in both $D$ and $U$.

We acknowledge that this proof is developed for a particular (still quite reasonable) sharing rule and we conclude that a sufficient condition to ensure that the $D$ firms do not interfere with the $U$ pricing is that their objectives are their own profits.

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25 This rule may be seen as a proxy to a Shapley value (Shapley 1953) solution for a corresponding one shot cooperative game on the sharing scheme.